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Bipartite flocking for multi-agent systems

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ABSTRACT

This paper addresses the bipartite flock control problem where a multi-agent system splits into two clusters upon internal or external excitations. Using structurally balanced signed graph theory, LaSalle's invariance principle and Barbalat's Lemma, we prove that the proposed algorithm guarantees a bipartite flocking behavior. In each of the two disjoint clusters, all individuals move with the same direction. Meanwhile, every pair of agents in different clusters moves with opposite directions. Moreover, all agents in the two separated clusters approach a common velocity magnitude, and collision avoidance among all agents is ensured as well. Finally, the proposed bipartite flock control method is examined by numerical simulations. The bipartite flocking motion addressed by this paper has its references in both natural collective motions and human group behaviors such as predator–prey and panic escaping scenarios.

1. Introduction

Flocking problems have attracted the interest of a lot of scientists from physics, mathematics, control engineering and biology due to its wide applications in mobile sensor networks, cooperative control of multiple robots or unmanned air vehicles (UAVs) [1–3]. The first well-known flocking model proposed in Reynolds' work [4] consists of three rules, i.e., (1) cohesion: attempt to stay close to nearby flockmates; (2) separation: avoid collisions with nearby flockmates; (3) alignment: attempt to match velocity with nearby flockmates. Based on Reynolds' three rules, flocking problems have been investigated from various perspectives [5–14].

However, the literatures mentioned above mainly focus on the connectivity preserving and velocity synchronization issues. Few existing results address the inverse problem, i.e., how to ensure a mutli-agent system to split into two oppositely moving clusters. Let's call this phenomenon a *bipartite flock* (a concept borrowed from [15]). Indeed, it is often encountered in biological flocks and human group behaviors that individuals have different motional targets or moving directions, which may be triggered by conflicting interests, differentiated opinions or external disturbances (like appearance of predators or obstacles) [16–22].

Conradt et al. [17,18] investigated the influential mechanism of intra-group interest conflicts on group behaviors, especially the velocity synchronization and fragmentation, i.e., separating into at least two clusters. It is found that group fragmentation happens upon large conflicts in the group. Couzin et al. [19] studied the collective selection of group direction when informed individuals differ in preference, and proposed that when the leaders' opinion divergent is low, the followers will obey the average preferred direction. Along increasing divergence, groups change from moving in the average preferred direction of all leaders to align to one of the two divergent preferred direction. Group fragmentation can find advantageous

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examples in abundant natural and social collective behaviors such as escaping panics [20] and natural predator–prey scenarios [21,22]. From engineering point of view, such as in robotic systems, when facing obstacles or 'enemies', the multi-robot group is expected to be separated into several clusters to avoid collisions. Therefore, it is of great importance and practical significance to study the bipartite control problem. However, the in-depth analytical work on bipartite mechanism is still on the way, and hence it is an urgent yet challenging task to design an effective control method for *bipartite flock*.

In [15] the bipartite consensus problem with conflicts of interest is investigated, and negative edge weights are used to represent the antagonistic relationship between two adjacent agents. Along this research line, consensus can be achieved with some mild conditions utilizing the properties of signed graphs [23] and structurally balanced networks [24,25]. In such networks, the positive (resp. negative) weights can represent cooperative (resp. competitive) relationship between each pair of adjacent agents, and each cycle of the graph has an even number of negative edges.

This motivates us to investigate the physical rule behind the abundant yet complex fragmentation phenomena of biological collective motions and social group behaviors. In this paper, the *bipartite flock* problems are investigated where the group of the multi-agent systems finally splits into two disjoint clusters moving with the same velocity magnitude but opposite directions. Meanwhile inter-agent collisions are avoided as well.

The remainder of the paper is organized as follows. Section 2 gives the formulation of the bipartite flocking problem and some necessary graph theory. Afterwards, the main results of bipartite flocking algorithms are introduced in Section 3. Section 4 presents numerical simulations to examine the effectiveness of the control developed in this paper. Finally, concluding remarks are drawn in Section 5.

2. Preliminaries and problem description

Throughout this paper, the following notations are used: $\mathbb{R}^m$ is an m-dimensional real vector space, I_m is an $m \times m$ identity matrix, and $\mathbf{1}_n = [1, \ldots, 1]^\top \in \mathbb{R}^n$. Besides, symbols $\cdot^\top$, $\|\cdot\|$ and $\otimes$ denote matrix transpose, Euclidean norm and the Kronecker product, respectively.

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, A\}$ denote a signed graph, where $\mathcal{V} = \{1, \ldots, n\}$ denotes the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set and $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the adjacency matrix of $\mathcal{G}: a_{ij} = \pm C \Longleftrightarrow (j,i) \in \mathcal{E}$, where C is a positive constant, otherwise $a_{ij} = 0$. We use $\mathcal{G}(A)$ to denote the signed graph corresponding to A. Graphs with self-loops will not be considered through this paper. A path $\mathcal{H}$ of $\mathcal{G}(A)$ is a concatenation of edges of $\mathcal{E}$: $\mathcal{H} = \{(k_1, k_2), (k_2, k_3), \ldots, (k_{p-1}, k_p)\} \subset \mathcal{E}$ in which all nodes $k_1, \ldots, k_p$ are distinct. A cycle $\mathcal{C}$ of $\mathcal{G}(A)$ is a path beginning and ending with the same node $k_p = k_1$. A cycle is positive (negative) if it contains an even (odd) number of negative edge weights: $a_{i_1,i_2} \ldots a_{i_p,i_1} > 0 (< 0)$. A graph is undirected if its corresponding adjacency matrix A is symmetric ($a_{ij} = a_{ji}$). We call the relationship between agents i and j is collaborative if $a_{ij} > 0$, otherwise competitive if $a_{ij} < 0$.

Consider an infinite sequence of nonempty, bounded and contiguous time intervals $[t_r, t_{r+1}), r = 0, 1, \ldots$, with $t_0 = 0$ and $t_{r+1} - t_r \leqslant T$ for some constant $T > 0$. In each interval $[t_r, t_{r+1})$, there is a sequence of subintervals

$$[t_{r_0}, t_{r_1}), [t_{r_1}, t_{r_2}), \ldots, [t_{r_{m_r-1}}, t_{r_{m_r}})$$

with $t_{r_0} = t_r$ and $t_{r_{m_r}} = t_{r+1}$ satisfying $t_{r_{j+1}} - t_{r_j} \geqslant \tau, 0 \leqslant j \leqslant m_r - 1$ for some integer $m_r \geqslant 0$. Let $\pi(t) : [0,\infty) \to \mathcal{P} = \{1, 2, \ldots, M\}$ be a piecewise-constant switching signal with continuous switching times $t_{00}, t_{01}, \ldots$ and $\mathcal{P}$ be the index set associated with the elements of $\mathcal{G}_p$. Given a constant $\tau > 0$, the interaction graph switches at t_{r_j} and does not change during each interval $[t_{r_j}, t_{r_{j+1}})$. Evidently, there are at most $M = \lfloor T/\tau \rfloor$ subintervals in each interval $[t_r, t_{r+1})$, where $\lfloor T/\tau \rfloor$ denotes the maximum integer no larger than T/τ.

In this paper, we consider each agent in the multi-agent systems has the following double-integrator dynamics:

$$\begin{cases} \dot{q}_i(t) = p_i(t), \\ \dot{p}_i(t) = u_i(t), \quad i \in \mathbb{N} := \{1, \ldots, n\}, \end{cases} \tag{1}$$

where $q_i(t), p_i(t), u_i(t) \in \mathbb{R}^m$ are the position, velocity states and the control force of agent i, respectively. For conciseness we denote

$$q_{ij} = q_i - q_j, \quad (i,j) \in \mathbb{E}, \quad \mathbb{E} := \{(i,j) | i,j \in \mathbb{N}, i \neq j\}$$

as the relative distance between two agents.

In this paper, each agent is assumed to be equipped with two onboard sensors: a position sensor and a velocity sensor. Moreover, it is assumed that all sensors work instantaneously. We assume that the interaction topologies for the position and velocity information flows are different.

The position interaction network, denoted as $\mathcal{G}_q = \{\mathbb{N}, \mathcal{E}_q(t)\}$, is assumed to be dynamic (time-varying) along $t \in (0, +\infty)$. Its fixed sensing radius is R and hence the neighboring set of agent i is denoted as $\mathcal{N}_i = \{j | \|q_i - q_j\| < R, (i,j) \in \mathbb{E}\}$. Meanwhile, the velocity interaction graph depends on the existence of virtual leaders, if there is no virtual leader in the multi-agent system then it is an undirected, connected and structurally balanced signed graph, denoted as $\mathcal{G}_p(A) = \{\mathbb{N}, \mathcal{E}_p, A\}$ with corresponding adjacency matrix $A = [a_{ij}]$. Otherwise, it is an undirected *jointly structurally balanced* signed graph, denoted as $\mathcal{G}_p^\pi(A^\pi) = \{\mathbb{N}, \mathcal{E}_p^\pi, A^\pi\}$ with a time-varying adjacency matrix $A^\pi = [a_{ij}^\pi]$.

It should be noted that there is no actual leader among agents, all agents play the same role. However, we can view the external reference signal as a virtual leader [6].

To provide our main results, we need the following definitions and lemmas: