Computers and Fluids 123 (2015) 218–234

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

Two spheres sedimentation dynamics in a viscous liquid column

S.M. Dash*, T.S. Lee

Department of Mechanical Engineering, National University of Singapore, 117576, Singapore

ARTICLE INFO

Article history:
Received 12 March 2015
Revised 9 September 2015
Accepted 3 October 2015
Available online 20 October 2015

Keywords:
Sphere sedimentation
DKT
IB-LBM
Experiments
Streamlines

ABSTRACT

In this paper, results of three-dimensional numerical simulation and limited experimental study are presented on the sedimentation of two inline spheres in a Newtonian fluid at a Reynolds number (Re) range of 10 to 60. The study is motivated by the experiment of [Fortes (1987)] [1], which shows that two settling spheres uniquely experience a non-linear wake interaction that captures the trailing sphere in the low pressure wake of the leading one and accelerates its motion to form a pair of kissing spheres, follows by tumbling and splitting apart before traveling with the same velocity. The present study aims to address some unanswered questions regarding surface pressure distributions, hydrodynamic forces and induced turning couples acting on the spheres during the process of Drafting-Kissing-Tumbling (henceforth refer to as DKT). The experiments were conducted in a vertical tank containing glycerine-water mixture of a predetermined viscosity, and the numerical study was performed using a coupled immersed boundary-lattice Boltzmann scheme. In addition to the vorticity field and pressure distributions during DKT, our numerical results indicate that the tumbling mechanism is influenced by the turning couple, which together with repulsive hydrodynamic forces originates vertical and lateral migrations of the spheres until they acquire a steady state alignment. The sense of the turning couple dictates whether the spheres would exhibit *normal* or *inverse* tumbling mechanism. Our results further show that normalised trajectories, settling velocities and hydrodynamics force coefficients of sedimenting spheres are independent of Re, at least for the range of values considered here.

1. Introduction

The study of hydrodynamic interactions between two settling particles in a Newtonian fluid is of relevance to multi-particle interactions encountered in applications such as, processing of mineral ores, transportation of settling slurries in a pipeline and dispersion of pollutants in rivers, seas or in atmospheres. One of the earliest theoretical work on the sedimentation behaviour of a pair of spheres in a viscous fluid was carried out by Smoluchowski [2,3], for small Re and small ratio of sphere radius to their centre spacing. The limitation of the theoretical analysis at high Re, leads numerous researchers to conduct experimental [1,4,5] and numerical investigations [6–9] to better understand the sedimentation of small clusters of spheres or 2D particles. One of the interesting observations arising from these studies is that for Re $>$ 1, as two equal sized spheres fall vertically one behind the other, the rear one experiences non-linear wake interactions of the leading sphere. As a result, the trailing sphere is "drag" into the wake of the leading sphere (also known as drafting) which subsequently leads to them touching momentarily (also known as kissing) before tumbling to break the unstable kissing equilibrium state and then separates away. This observation raises two important questions: *(1) What are the driving hydrodynamic forces or couples that are responsible for the tumbling mechanism? (2) How do the separated settling spheres behave after the tumbling action?*

In an attempt to answer the first question, Hu et al. [5] compared the tumbling mechanism of the spheres to the settling of a long body in a Newtonian fluid. They reasoned that the fall of the long body experiences a turning couple induced by pressure distributions, stagnation and separation points that cause its broad side to align perpendicularly to the stream, which has close similarity to the tumbling action of the kissing spheres. Although these two cases may qualitatively exhibit some degree of similarities, they differ in the magnitude of the forces and turning couple on the spheres due to the fact that tumbled spheres may separate from each other, which is distinctly different from the fixed mass or geometry of the long body. In later studies [10,11] the tumbling action in Newtonian and viscoelastic fluids is attributed to the competition between inertia and normal stress terms that leads to the production of repulsive or attractive force among the spheres. They believed that the nature of this force, repulsive or otherwise, defines their tumbling action. To better understand how these forces are generated in the Newtonian fluid, numerous researchers [12–14] performed numerical simulations on three dimensional flow past two identical stationary spheres held fixed relative to each other and with their connecting centreline normal

* Corresponding author. Tel.: +6584341457.
E-mail address: mpedsm@nus.edu.sg, sunilmanohardash@gmail.com (S.M. Dash).

http://dx.doi.org/10.1016/j.compfluid.2015.10.003
0045-7930/

to the uniform free-stream. They observed that the wake structures for small centre spacing differ significantly from that for large centre spacing, and postulated that this generates lift, drag and moment that led to repulsion or attraction among the spheres. In another similar study, Kim et al. [15] conducted numerical simulations on the unsteady interaction of an advected cylindrical vortex tube and a stationary sphere where the sphere experiences drag force. They have attributed the generation of this drag force to the distinctive shear stress distribution originated from the induced velocity of the upwash and downwash region. Although the study of fixed sphere wake interaction is useful in the characterisation of particle-laden flow, the analogy is incomplete without taking into consideration of the additional inertia effects experienced by the freely falling spheres. Moreover, the unsteady nature of the hydrodynamic forces during tumbling action may not be clearly identified in the simple flow past the tandem stationary spheres [12,14], or in an advected vortical structure and sphere interaction [15]. We believe that a deeper understanding of DKT action can be made if we can identify temporal evolution of the unsteady hydrodynamic force on the settling spheres. The desire to do this motivated us to carry out the present study.

Another motivation is related to the above-mentioned second question regarding the behaviour of separated settling spheres after the tumbling action. Hu et al. [5] observed that in a Poiseuille like flow condition the tumbled spheres attain a steady equilibrium position when the trailing sphere locks itself in the wake of the leading one, and thereafter move together with certain centre alignment analogous to the flying bird flock. In a separate numerical study, Singh et al. [16] investigated the stable arrangement of two dimensional particles (or two dimensional rods) in periodic arrays subjected to a cross flow. They noted that the arrays of the particles are stable only when they have equal centre spacing and are arranged in a line normal to the flow direction, otherwise the particles are prone to wake interactions and drafting. Furthermore, in analogous studies [17,18] on rising of the two spherical bubbles, suggest that depending on their initial separation the freely moving bubbles may experience potential (venturi) effect, viscous correction and significant wake effects when the second bubble travels in the wake of the first bubble. Also, there exists an equilibrium separation distance for the tandem bubble moving inline. However, these spherical bubbles are subjected to surface deformation and surface contamination which affects their equilibrium separation and hence, these studies differ from the present work on the sedimentation the rigid spheres. Moreover, due to lack of experimental and numerical evidence and analysis on the stable equilibrium configuration of the freely falling tandem spheres has lead us to conduct the present three-dimensional (3D) study. Here, we have focused our attention on 3D trajectories of the tumbled spheres, including their lateral, vertical migrations and subsequent steady stable alignment as a function of Re.

For the experimental aspect of the present study, we have fabricated an apparatus to perform qualitative visualisation of the spherical particles sedimentation for Re ranging from 10 to 60. As pressure field and hydrodynamic force on moving sphere are difficult to measure experimentally, the numerical study is used to bridge this gap. Here, the numerical simulations are carried out using our recently developed flexible forcing immersed boundary and lattice Boltzmann model (IB-LBM) [19–21]. The IB method is selected to reduce computational time and effort by avoiding time-dependent grid regeneration for the moving particle [22]. Concurrently, LBM acts as an efficient alternative over traditional Navier–Stokes (NS) solver where the pressure Poisson's equation is not solved and the inherent parallelization facility helps to accelerate the simulation process [23]. More detailed information on IB and LBM can be found in the excellent review articles [24–26] by the authors of Ref. 24–26.

This paper is organized as follows. Section 2 presents the numerical scheme adopted here and Section 3 discusses the experimental setup and methodology. Validation of the numerical code is given in Sections 4, and 5 presents the problem definition of two spherical particle sedimentation and the corresponding flow regimes. Conclusions are drawn in Section 6.

2. Numerical method

As noted earlier, the numerical scheme adopted here is based on the flexible forcing IB-LBM. The scheme was developed by the authors and has been employed to study particle sedimentation (see Ref. [19–21]). Details of the scheme can be found in the above cited references. Only the essential features, such as governing equations, kinematics of the sphere motion are discussed in the following section.

2.1. Flexible forcing IB-LBM

For three-dimensional viscous, incompressible and unsteady flows, the governing equations in the IB-LBM with discrete body force F_α can be written as [27,28],

$$f_\alpha(\mathbf{x}_{ijk} + \boldsymbol{e}_\alpha\delta t, t+\delta t) = f_\alpha(\mathbf{x}_{ijk}, t) - \frac{1}{\tau}\left(f_\alpha(\mathbf{x}_{ijk}, t) - f_\alpha^{eq}(\mathbf{x}_{ijk}, t)\right) + F_\alpha\delta t, \quad (1)$$

$$f_\alpha^{eq}(\mathbf{x}_{ijk}, t) = \rho W_\alpha\left[1 + \frac{\boldsymbol{e}_\alpha\cdot\mathbf{u}}{c_s^2} + \frac{(\boldsymbol{e}_\alpha\cdot\mathbf{u})^2 - (c_s|\mathbf{u}|)^2}{2c_s^4}\right], \quad (2)$$

$$F_\alpha = \left(1 - \frac{1}{2\tau}\right)W_\alpha\left(\frac{\boldsymbol{e}_\alpha - \mathbf{u}}{c_s^2} + \frac{\boldsymbol{e}_\alpha\cdot\mathbf{u}}{c_s^4}\boldsymbol{e}_\alpha\right)\cdot\mathbf{f}, \quad (3)$$

$$\rho\mathbf{u} = \sum_\alpha \boldsymbol{e}_\alpha f_\alpha + \frac{1}{2}\mathbf{f}\delta t, \quad (4)$$

where $f_\alpha(\mathbf{x}_{ijk}, t)$ and $f_\alpha^{eq}(\mathbf{x}_{ijk}, t)$ are the density distribution functions and its corresponding equilibrium states at computational time t. In the present 3D simulations, a standard D3Q15 lattice velocity model [29] is used, where $f_\alpha(\mathbf{x}_{ijk}, t)$, $f_\alpha^{eq}(\mathbf{x}_{ijk}, t)$ and F_α at Eulerian nodes ($\mathbf{x}_{ijk}$) have 15 components along the lattice directions $\alpha = 0, 1, 2, \ldots, 14$ (see Ref. [20,29]). The discrete lattice velocities $\boldsymbol{e}_\alpha$ with their Cartesian components: $e_{\alpha x}$, $e_{\alpha y}$, $e_{\alpha z}$ and respective weighting co-efficient W_α of D3Q15 model are shown in Table 1. Here, c is the lattice speed, which is defined as $c = \delta x/\delta t$. The present study uses the lattice units of mesh, $\delta x = 1$ and time step size $\delta t = 1$. The sound speed is defined as, $c_S = c/\sqrt{3}$. τ is the single relaxation time related to kinematic viscosity ν as, $\nu = (2\tau - 1)/6$, and $\mathbf{f}$ is the Eulerian force density which is distributed from the Lagrangian boundary force density.

The included force density $\mathbf{f}$ in Eqs. (3) and (4) can be derived and updated using either an explicit [30,31] or an implicit [27,28,32,33] scheme so as to satisfy the no-slip boundary condition. One of the drawbacks of the explicit schemes is that the pre-defined force density may produce unsatisfactory no-slip condition that leads to non-physical streamline penetration into the solid object. In contrast, the force density in the implicit scheme is calculated using the implicit velocity correction that satisfies the exact no-slip condition. In the implicit schemes, the inherent matrix mathematics may limit the numerical schemes applicability to simple 2D cases only [27,28,32]. For complex 3D problems, increased matrix size demands higher computational resources and time to perform simple matrix operation and inversion. Recently, we have proposed [19,20] an alternate iterative scheme that avoids the matrix formulation and uses a single Lagrangian velocity correction to satisfy the accurate no-slip boundary condition as shown in Eq. (5).

$$\mathbf{U}_B^{pd}\left(\mathbf{X}_B^p, t\right) = \sum_{ijk}\mathbf{u}\left(\mathbf{x}_{ijk}, t\right)D\left(\mathbf{x}_{ijk} - \mathbf{X}_B^p\right)\Delta x\Delta y\Delta z + \delta\mathbf{U}_B^p\left(\mathbf{X}_B^p, t\right), \quad (5)$$

where $\mathbf{U}_B^{pd}$ is the desired boundary velocity and $\delta\mathbf{U}_B^p$ is the boundary velocity correction. Following the above correction principles, one