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Improved theory of generalized meteo-ballistic weighting factor functions and their use

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Abstract

It follows from the analysis of artillery fire errors that approximately two-thirds of the inaccuracy of indirect artillery fire is caused by inaccuracies in the determination of the meteo parameters included in fire error budget model. Trajectories calculated under non-standard conditions are considered to be perturbed. The tools utilized for the analysis of perturbed trajectories are weighting factor functions (WFFs) which are a special kind of sensitivity functions. WFFs are used for calculation of meteo ballistic elements μ_B (ballistic wind w_B, density ρ_B, virtual temperature τ_B, pressure p_B) as well. We have found that the existing theory of WFF calculation has several significant shortcomings. The aim of the article is to present a new, improved theory of generalized WFFs that eliminates the deficiencies found. Using this theory will improve methods for designing firing tables, fire control systems algorithms, and meteo message generation algorithms.

Keywords: Exterior ballistic; Non-standard projectile trajectory; Perturbation; Meteoconditions; Sensitivity function; (Generalized) weighting (factor) function (curve); (Generalized) effect function; Norm effect

1. Introduction

1.1. Motivation

It follows from the analysis of artillery fire errors, e.g. [1,2], that approximately two-thirds of the inaccuracy of indirect artillery fire is caused by inaccuracies in the determination of meteo parameters included in the error budget model [1]. Consequently, it is always important to pay close attention to the problems of including the actual meteo parameters in ballistic calculations [3]. The following meteo parameters μ are primarily utilized: Wind vector w, air pressure p, virtual temperature τ, and density ρ [2–6].

This paper deals only with problems relating to unguided projectiles without propulsion system for the sake of lucidity of the solved problems.

1.2. Weighting functions – basic information

The most important information about the influence of meteo parameters (and not only them) on the trajectory of an unguided projectile is included in the relevant weight or weighting functions [2,7–10].

List of notation

μ	met parameter (element)
$\mu(y)$	real or measured magnitude of met parameter μ in height y
$r(\mu)$	weighting factor function (curve, WFF)
Q_P, Q_{CP}	effect function
$\mu_{STD}(h)$	met parameter standard course with the height h
$\Delta\mu(y)$	absolute deviation of met element μ in height y
$\delta\mu(y)$	relative deviation of met element μ in height y
$\Delta\mu_B$	absolute ballistic deviation of ballistic element μ_B
$\delta\mu_B$	relative ballistic deviation of ballistic element μ_B

The basis for the derivation of the weighting functions is perturbation theory [11].

We are interested in the exercise of the perturbation theory in dynamical systems theory, primarily in the control theory of dynamical systems [12,13]. It is exercised especially in the exploration of stability and sensitivity [12,13]. The most

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widespread variant of the perturbation theory is the simplest one – the first-order perturbation theory. Its most important basis is the linearization of all requisite non-linear functions and equations [11–13].Unless otherwise specified, the following information refers to this theory.

A special subset of controlled systems is comprised of aerospace vehicles, i.e., aircrafts, space vehicles, rockets, space shuttles, guided missiles and spinning and non-spinning “unguided” projectiles with/without terminal guidance and Magnus rotors [4–6,14].

The state equations of non-linear dynamical systems have then the form

$$\boldsymbol{x}'(t) = \boldsymbol{X}[t, \boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{d}(t), \boldsymbol{\alpha}(t)], \quad \boldsymbol{x}(0) = \boldsymbol{x}_0$$

$$\boldsymbol{y}(t) = \boldsymbol{Y}[t, \boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{d}(t), \boldsymbol{\alpha}(t)] \tag{1}$$

where $\boldsymbol{x}$ is state variable vector, $\boldsymbol{u}$ is input control variable vector, $\boldsymbol{d}$ is input disturbance variable vector, $\boldsymbol{\alpha}$ is parameters vector, $\boldsymbol{y}$ is output variable vector.

The perturbation theory is used for transformation of these equations into their linearized form (first example) [5–9]. The linearized state equations have for example the form

$$\boldsymbol{x}'(t) \approx \boldsymbol{A}(t, \boldsymbol{\alpha}(t)) \cdot \boldsymbol{x}(t) + \boldsymbol{B}(t, \boldsymbol{\alpha}(t)) \cdot (\boldsymbol{u}(t) + \boldsymbol{d}(t)), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0$$
$$\boldsymbol{y}(t) \approx \boldsymbol{C}(t, \boldsymbol{\alpha}(t)) \cdot \boldsymbol{x}(t) + \boldsymbol{D}(t, \boldsymbol{\alpha}(t)) \cdot (\boldsymbol{u}(t) + \boldsymbol{d}(t)) \tag{2}$$

For the analysis of dynamical systems, it is interesting to observe changes of the system properties while some parameters $\boldsymbol{\alpha}(t)$ change; the parameters are then often denoted as influence quantities. We speak of differential sensitivity analysis of the control system or of sensitivity of a system to parameter variations. The perturbation theory is used again for linearization of Eq. (2) relative to parameters $\boldsymbol{\alpha}$. We obtain a sensitivity model of the (linearized) dynamical system, for instance in the form [12,13,15,16]

$$\boldsymbol{\lambda}'_i(t, \boldsymbol{\alpha}) \approx \boldsymbol{A}(t, \boldsymbol{\alpha}) \cdot \boldsymbol{\lambda}_i(t, \boldsymbol{\alpha}) + \frac{\partial \boldsymbol{A}}{\partial \alpha_i} \cdot \boldsymbol{x}(t, \boldsymbol{\alpha}) + \frac{\partial \boldsymbol{B}}{\partial \alpha_i} \cdot (\boldsymbol{u}(t) + \mathbf{d}(t))$$
$$\boldsymbol{\lambda}_i(0, \boldsymbol{\alpha}) = \boldsymbol{\lambda}_{0,i} \tag{3}$$
$$\boldsymbol{\eta}_i(t, \boldsymbol{\alpha}) \approx \boldsymbol{C}(t, \boldsymbol{\alpha}) \cdot \boldsymbol{\lambda}_i(t, \boldsymbol{\alpha}) + \frac{\partial \boldsymbol{C}}{\partial \alpha_i} \cdot \boldsymbol{x}(t, \boldsymbol{\alpha}) + \frac{\partial D}{\partial \alpha_i} \cdot (\boldsymbol{u}(t) + \mathbf{d}(t))$$

where

$$\boldsymbol{\lambda}_i\colon \boldsymbol{\lambda}_i(t, \boldsymbol{\alpha}) = \frac{\partial \boldsymbol{x}}{\partial \alpha_i}, \quad \boldsymbol{\eta}_i : \boldsymbol{\eta}_i(t, \boldsymbol{\alpha}) = \frac{\partial \boldsymbol{y}}{\partial \alpha_i}, \quad i = 1, 2, \ldots, n$$

are the absolute sensitivity functions. The absolute sensitivity functions of the output variables $\boldsymbol{\eta}_i$ are especially important for the practice. Non-dimensional Bode sensitivity functions are often used [12,13,15,16].

The perturbation theory is used in this second case for finding linearized relations between changes of system parameters $\Delta\boldsymbol{\alpha}$ and corresponding changes of the output variables $\Delta\boldsymbol{y}$, which are represented by the sensitivity functions $\boldsymbol{\eta}_i$ ($\boldsymbol{\eta}_i \approx \Delta\boldsymbol{y}/\Delta\alpha_i$, $i = 1, 2, \ldots, n$) and which can be expressed consecutively through the use of the corresponding transfer functions [12,13].

Standard test functions for the control variables $\boldsymbol{u}(t)$ and the disturbance variables $\boldsymbol{d}(t)$ are used for the analysis of properties of the systems that are described by Eqs. (1), (2) and (3). The unit impulse is usually used, and also the unit step, the function sine and/or cosine, etc. [12,13].

Such a procedure is not sufficient for analyses of movements of aerospace vehicles, so it is customary to use reference trajectories and maneuvers, respectively, which represent the typical maneuvers of a given type of aerospace vehicle [4–6,14].

Moreover, it is necessary to differentiate whether reference maneuvers are pursued under standard conditions or perturbed conditions.

Standard conditions are defined contractually and determine the standard/normal values of the parameters respectively $\boldsymbol{\alpha}_{STD}(t)$, for instance, parameters of the standard atmosphere are considered. The reference maneuvers under standard conditions are utilized for the basic analysis of aerospace vehicle properties, Eqs. (1) or (2) are used withal ($\boldsymbol{d}(t) = 0$).

The reference maneuvers under perturbed conditions ($\boldsymbol{d}(t) \neq 0$, respectively $\boldsymbol{\alpha}(t) = \boldsymbol{\alpha}_{STD}(t) + \Delta\boldsymbol{\alpha}(t)$) serve for consequential analyses of stability or robustness of flight control; Eq. (3) are used together with Eqs. (1) or (2).

The reference maneuver under standard conditions in the exterior ballistics of unguided projectiles is represented just by the standard projectile trajectory, and the reference maneuver under perturbed conditions is identical to the relevant perturbed projectile trajectory.

As mentioned above, corresponding sets of transfer functions are referred to Eqs. (2) and (3); their equivalent in the time domain is the convolution operation represented by the convolution integral. Two functions *f* and *g* figure in the convolution integral. The functions f and g have a special significance in the control theory of dynamical systems. The function *f* represents a generalized input variable $z_m(t)$, $m = 1, 2, \ldots$ (respectively $u_j(t)$, $j = 1, 2, \ldots$ and $d_k(t)$, $k = 1, 2, \ldots$ and $\Delta\alpha_i(t)$, $i = 1, 2, \ldots$) and the function $g_{m,l}(t)$ is the weighting function that corresponds with the relevant transfer function. The integral value then corresponds to the system response y_l, $l = 1, 2, \ldots$, to the excitation by the input variable [12,13].

The weighting functions $g_{ml}(t)$ are impulse-response functions [12,13], i.e. responses of the dynamical system to the special excitation by impulse function $z_m(t) = z_{m0} \cdot \delta(t - t_p)$, where z_{m0} is the excitation amplitude and $\delta(t - t_p)$ is the Dirac delta function. The weighting function then has the form

$$g_{ml}(t - t_p) = (M_{mn} / z_{m0}) \cdot \gamma_{ml}(t - t_p) \tag{4}$$

where

t_p is the moment of the impulse occurrence,
$\gamma_{ml}(t - t_p)$ is the normed form of the weighting function and
M_{mn} is the relevant norm.