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Randomness and arbitrary coordination in the reactive ultimatum game

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ABSTRACT

Darwin's theory of evolution - as introduced in game theory by Maynard Smith - is not the only important evolutionary aspect in an evolutionary dynamics, since complex interdependencies, competition, and growth should be modeled by, for example, reactive aspects. In the ultimatum game, the reciprocity and the fifty–fifty partition seems to be a deviation from rational behavior of the players under the light of Nash equilibrium. Such equilibrium emerges, for example, from the punishment of the responder who generally tends to refuse unfair proposals. In the iterated version of the game, the proposers are able to improve their proposals by adding a value thus making fairer proposals. Such evolutionary aspects are not properly Darwinian-motivated, but they are endowed with a fundamental aspect: they reflect their actions according to value of the offers. Recently, a reactive version of the ultimatum game where acceptance occurs with fixed probability was proposed. In this paper, we aim at exploring this reactive version of the ultimatum game where the acceptance by players depends on the offer. In order to do so, we analyze two situations: (i) mean field and (ii) we consider players inserted within the networks with arbitrary coordination. We then show that the reactive aspect, here studied, thus far not analyzed in the evolutionary game theory literature can unveil an essential feature for the convergence to fifty–fifty split. Moreover we also analyze populations under four different polices ranging from a highly conservative to a moderate one, with respect to the decision in changing the proposal based on acceptances. We show that the idea of gaining less more times added to the reciprocity of the players is highly relevant to the concept of "healthy" societies population bargaining.

1. Introduction

Game theory analyzes several important aspects of the Economical and Biological sciences such as bargaining, cooperation and other social features. The theory plays an important role in explaining the interaction between individuals in homogeneous and heterogeneous populations, with or without spatial structure, in which agents negotiate/combat/collaborate via certain protocols. The full understanding of cooperation between individuals as an emergent collective behavior remains an open challenge [1–3]. In this context, bargaining is an important feature has called attention of many authors: two play-

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ers must divide an amount (resources, money, food, or other interesting quantity) and the disagreement (or no agreement) between them in a given deal could mean that both lose something. This dilemma motivates a simple game that mimics the bargaining between two players - the ultimatum game.

In this game, firstly proposed by Güth et al. [4], one of the players proposes a division (the proposer) and the second player (the responder) can either accept or reject it. If the responder (the second player) accepts it, the values are distributed according to the division established by the proposer. Otherwise, no earning is distributed to both players.

Real situations in western societies suggest that unfair proposals are refused for either fairer or even more selfish amounts. However some isolated societies as Machiguenga localized at Peruvian Amazon seem to show a behavior opposed to such fact, which suggests a more altruistic behavior [5]. On the other hand, scientists have studied and simulated artificial societies where players confront each other according the ultimatum game protocol. In order to consider a simple evolutionary probabilistic model where unsatisfactory proposals are refused, in this paper we propose to study a model where accepting depends on proposal.[1] An interesting study [6] shows that humans (children) and chimpanzees display similar preferences regarding reward division. Moreover both (humans and chimpanzees) preferred the selfish option with passive partners in a version of ultimatum game known as the dictator game.

Although it is rationally better for the responder to accept any offer, offers below one third of the available amount to be shared are often rejected [7]. The responder punishes the proposer up to the balance between proposal and acceptance in the iterated game. In general, values around a half of the total amount are accepted [7,8]. Other interesting experimental results suggest that high-testosterone men reject low offers in the ultimatum game [9]. Nowak et. al [10] showed that the evolution of fairness, similarly to the evolution of cooperation, is linked to reputation by considering a simple memory mechanism: fairness will evolve if the proposer can obtain some information on what deals the responder has accepted in the past. Other authors have shown that spatial structures such as regular and complex networks facilitate the emergence of fairness (see, for example [11,12].

Our contribution goes precisely along this line of research. In this manuscript, we extend the memory-1 model proposed by one of the authors in [13] that considers the acceptance with fixed probability, by putting this probability variable and assigning the offer O_t, at time t, that is a number belonging to [0, 1] and performing the game in graphs with arbitrary homogeneous and heterogeneous coordination.

In this reactive and iterated version of the ultimatum game, the players are able to correct their offers by adding/subtracting an amount to the offers in order to make fairer proposals. Such mechanisms, which we assume are an essential ingredient for the convergence to fifty–fifty partitions seems to be discarded in typical evolutionary game theory based on probabilistic Darwinian copies. By performing a detailed study, we investigate the game both analytically and via Monte Carlo (MC) simulations under four different policies about the increase or decrease of the offer under different levels of greed. Moreover, we present results about temporal correlations in the model with fixed probability for a suitable comparison with the model where the offer is time-dependent.

The remainder of paper is organized as follows. Next, we define the reactive model and its mean-field approximation. Then, we show how the model can be run in networks with arbitrary coordination. In Section 2 we present the first part of our results corresponding to the mean-field approximation. In Section 5 we present the results for the game with arbitrary coordination via equation integrations. Particularly for $k = 4$ we explore the randomness effects by considering MC simulations in small world networks. A general and analytical formula is obtained for the stationary average offer and a complete study of the fluctuations and distribution of the payoffs are performed considering homogeneous and heterogeneous populations. Then we present a comparative analysis between mean-field and the model on networks. Finally, we conclude and comment on the relevance of the reactive ultimatum game, in particular on the experimental evidence of the effect of fairer offers in different international societies.

2. Modeling and mean-field approximation: analyzing the correlations

In the reactive ultimatum game, when a player (proposer) performs an offer $O_t \in [0, 1]$ at time t, it can be accepted or rejected by the other player (i.e. the responder). Let us think that such acceptance occurs with probability p_t. Let us consider two simple situations:

1. $p_t = p$ fixed, and does not change along time;
2. $p_t = O_t$, i.e., the acceptance occurs with higher probability as the offer is more generous.

When the offer is rejected it will take the proposer to change its expectations increasing its proposed offer ϵ. On the other hand when it is accepted the proposer decreases its proposal by a quantity ϵ. Here ϵ is a rate of offer change. We can consider the mean-field regime as the average under all different time series of parameters of two players interacting according to a dynamics. We also can imagine it as parameters averaged by the different players in a large population, where the players interact at each time t (denoted by authors in Refs. [14,15] as one 'turn') by pairs composing a perfect matching with N players (for the sake of simplicity N is an even number) randomly composed. In this pairing, no player is left out of the game, with each individual playing once by turn, by construction.

[1] This game scenario is common and expected in real situations, at least in western societies, illustrated even when children negotiate chocolate coins (see e.g. this video https://www.youtube.com/watch?v=YXfEv-xEWtE).