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Explicit exact solutions for (2 + 1)-dimensional Boiti–Leon–Pempinelli equation ☆

Wei-Gui Feng, Kai-Ming Li, Ya-Zhou Li, Chang Lin *

College of Physics and Electronics Engineering, Northwest Normal University, Lanzhou 730070, China

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ABSTRACT

In this paper, we construct explicit exact solutions for the coupled Boiti–Leon–Pempinelli equation (BLP equation) by using a extended tanh method and symbolic computation system *Mathematica*. By means of the method, many new exact travelling wave solutions for the BLP system are successfully obtained. the extended tanh method can be applied to other higher-dimensional coupled nonlinear evolution equations in mathematical physics.

1. Introduction

The investigation of exact travelling wave solutions to nonlinear evolution equations (NLEEs) plays an important role in the study of nonlinear physical phenomena. In the line with development of computerized symbolic computation, much work has been focused on the various extensions and application of the known algebraic methods to construct the solutions of NLEEs [1–19]. Recently, Wazwaz firstly proposed the extended tanh method in [1,2] and successfully construct new and more general rational formal solutions. The integrability of the BLP system [3], soliton-like, multisoliton-like [4], some special soliton interaction behaviors [5–8], and some exact solutions [9,10] have been discussed. In the paper, we used the extended tanh method to investigate the coupling Boiti–Leon–Pempinelli system [3–10]

$$\begin{aligned} u_{ty} &= (u^2 - u_x)_{xy} + 2v_{xxx}, \\ v_t &= v_{xx} + 2uv_x \end{aligned} \tag{1}$$

and obtained some new exact travelling solutions.

The organization of the paper is as follows. In Section 2, a extended tanh method is presented. In Section 3, we investigate the BLP system using the extended tanh method and obtain some kink soliton solutions and travelling solutions. Finally, in Section 4, some conclusions are given.

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* Corresponding author.
E-mail addresses: the.red.rose@163.com (W.-G. Feng), linchangzhang@tom.com (C. Lin).

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2. The extended tanh method

We now present briefly the main steps of the extended tanh method that will be applied:

Step 1. *We first consider a given NLEE system with some physical fields $u_i(x,y,t)$ in three variables x, y, t*

$$P_i(u_i, u_{it}, u_{ix}, u_{iy}, u_{itt}, u_{ixt}, u_{iyt}, u_{ixx}, u_{iyy}, u_{ixy}, \ldots) = 0, \tag{2}$$

by using the wave transformation

$$u_i(x,y,t) = U_i(\xi), \quad \xi = k(x + ly - \lambda t), \tag{3}$$

where k, l, λ are constants to be determined later. Then the nonlinear partial differential equation (2) is reduced to a nonlinear ordinary differential equation (ODE)

$$Q_i(u_i, u_i', u_i'', u_i''', \ldots) = 0. \tag{4}$$

Step 2. *If all terms of the resulting ODE(4) contain derivatives in ξ, then by integrating this equation, and by considering the constant of integration to be zero, we obtain a simplified ODE.*

Step 3. *We then introduce a new independent variable*

$$Y = \tanh(\mu\xi), \tag{5}$$

that leads to the change of derivatives

$$\frac{d}{d\xi} = \mu(1 - Y^2)\frac{d}{dY}, \quad \frac{d^2}{d\xi^2} = -2\mu^2 Y(1 - Y^2)\frac{d}{dY} + \mu^2(1 - Y^2)^2\frac{d^2}{dY^2} \tag{6}$$

Step 4. *The extended tanh method admits the use of the finite expansion*

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k + \sum_{k=1}^{M} b_k Y^{-k}, \tag{7}$$

where M is a positive integer, in most cases, that will be determined. Expansion (7) reduces to the standard tanh method for $b_k = 0, 1 \leqslant k \leqslant M$. To determine the parameter M, we usually balance the linear terms of highest order in the resulting equation with the highest-order nonlinear terms.

Step 5. *Substituting (7) into the ODE (4) results in an algebraic equation in powers of $Y^{\pm i}$. We then collect all coefficient of powers of $Y^{\pm i}$ in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters a_k, b_k, $(k = 0, \ldots, M)$, μ, and c.* Having determined these parameters we obtain an analytic solution $u(x,t)$ in a closed form.

3. Exact solutions of the Boiti–Leon–Pempinelli system

Using the wave variable $u(x,y,t) = u(\xi)$ and $v(x,y,t) = v(\xi)$ where $\xi = k(x + ly - \lambda t)$ carries the BLP system Eq. (1) into a system of ODEs

$$\begin{aligned} &-\lambda l u'' = 2l(uu')' - klu''' + 2kv''', \\ &-\lambda v' = kv'' + 2uv', \end{aligned} \tag{8}$$

where by integrating twice the first equation we find

$$v' = \frac{klu' - lu^2 - \lambda lu}{2k}. \tag{9}$$

Integrating (9) and taking the integral constant be zero, we obtain

$$v = \frac{1}{2}lu - \frac{1}{2k}\int (lu^2 + \lambda lu)d\xi. \tag{10}$$

Substituting Eq. (9) into the second equation of Eq. (8), we obtain

$$k^2u'' - 2u^3 - 3\lambda u^2 - \lambda^2 u = 0. \tag{11}$$

Balancing u'' with u^3 in Eq. (11) gives $M = 1$. According to Eqs. (10) and (11), we will determine $u(x,y,t)$ and $v(x,y,t)$.

For Eq. (11), the extended tanh method admits the use of

$$u(\xi) = a_0 + a_1 Y + b_1 Y^{-1}. \tag{12}$$