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Coupled radiative and conjugate heat transfer in participating media using lattice Boltzmann methods

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ABSTRACT

In the past, lattice Boltzmann methods (LBM) have been extensively developed for momentum and energy transport in single-phase and multi-phase fluid systems. Recently, LBM based algorithms have been developed and applied to fundamental Radiative Transport Equations (RTE), including radiation–material interactions and were found very convenient to model radiative energy exchange between radiation and material medium. This work advances the development of Lattice Boltzmann Equations (LBE) for radiative transport by integrating them with existing LBEs for energy and momentum transport for solving multi-physics problems. The multi-physics example problems of thermal energy transport where radiation, conduction and convection all are considered as important modes are modeled via this integrated LBM. These integrated LBM models are used to solve one and two dimensional problems, and highlight the advantage of this approach for solving multi-physics problems in a single framework. First example involves modeling radiative and conductive heat transfer in one-dimensional slab using LBM. The numerical results are compared with existing benchmark P_1 solutions. Next example is the simulation of two-dimensional radiative porous burner with hot walls. This problem is simulated with two numerical models: a homogenous porous media and a heterogenous model of packed obstacles which have differential scattering and absorption interactions. The homogenous model uses analytical velocity field and provides a simpler approach, but has limitations in providing detailed analysis. In heterogenous model velocity field, temperature field and radiation field are computed with a set of coupled LBEs. Fluid flowing through heterogenous porous media undergoes conjugate heat exchange with obstacles and also interacts with isotropic incident radiation. These two-dimensional example cases with different material properties are solved with D_2Q_{16} LBE template.

1. Introduction

The advancement in the field of high temperature materials over the period of last two decades has potential to completely revolutionize the energy industry with the design of high efficiency systems. With the higher temperature in the energy system applications, radiation heat transfer becomes equally important along with conduction and convection modes of heat transfer. Therefore, the radiative mode of heat transfer with conduction and convection has growing significant practical importance in many engineering applications [1–3]. Some of these industrial applications include the manufacturing of glass, design of insulating material, weather forecasting, porous burners, solar collectors, high temperature nuclear reactors etc. In the past, most of the commercial and academic codes and calculations for the multi-mode heat transfer treated radiative transport using surface to surface boundary conditions. There are very few numerical codes which solve the radiative transport within the participating media where conduction and convection calculations are performed. The radiative portion in a multi-physics set-up requires tremendous computational effort due to the complexity of radiation interaction within a medium which is directionally sensitive process.

The RTE solutions are computationally expensive due to the fact that at each point of evaluation all the radiative sources within the medium have to be accumulated from their initial starting point to the point of interest. This recursive nature of the RTE is compounded by the fact that not all radiation in the media has the same energy, thus another dimension must be accounted for during the simulation. Moreover, radiation transport from a point to the surrounding regions is highly directional dependent therefore higher degree of angular resolution is required in computations [4–11]. This implies there are additional independent variables i.e. energy level and angular direction. Due to larger number of in-

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dependent variables for RTEs as compared to standard continuum mechanics equations, Monte Carlo methods are preferred over deterministic solvers [12]. But Monte Carlo methods are not so convenient for conventional continuum mechanics problems such as fluid mechanics or convection–diffusion of scalar quantities. Therefore, a new approach or framework is required for multi-physics problems with radiation transport as one of the significant physical processes. Some new multi-physics problems involving radiation–material interactions at mesoscale such as photo-thermal heating and mixing and optical tweezers have not been understood quantitatively as they required mutually coupled modeling tool between radiation transport and particle transport. LBM has been proposed as a new method for solving multi physics problems involving RTE. However, most of the effort was focused on coupling Discrete Ordinates Method (DOM) or other conventional deterministic methods for solving RTEs with the LBM based fluid transport or convection–diffusion solvers [13–16]. Recently, LBM based algorithms have been developed to solve RTE problems [17–22]. This paper proposes the use of those LBM algorithms for solving coupled multi physics examples. The problems shown in this paper extend the application of LBM to non-linear temperature driven coupled conduction–radiation and coupled convection–radiation in different one-dimensional and two-dimensional geometries.

This paper is divided into three main sections. The next section discusses theoretical background on RTE, its coupling with material energy transport and adoption of LBM to solve those coupled interactions. The subsequent section describes problem set-up, numerical details and results for conduction–radiation and convection–radiation examples solved using the LBEs. In this section, thermal energy transport in a simplified version of porous burner is modeled with convection, conduction, and radiation simultaneously. In the last section final conclusions are provided.

2. Mathematical formulation

2.1. Radiation–material energy interactions

The radiation–material interaction can be considered as equilibrium interaction, where radiation and material energies reach an instantaneous equilibrium, or non-equilibrium interaction, when there is a lag between radiative energy and material energy. The non-equilibrium interaction of radiation and material energies is modeled by coupling the radiation transport equation (RTE) with the material energy balance equation [23]. Assuming homogenous materials and isotropic scattering, the set of governing equations for non-equilibrium radiative transfer can be written as

$$\begin{aligned}\frac{\partial I(\mathbf{r},\mathbf{\Omega},t)}{\partial t}+\mathbf{\Omega}.\nabla I(\mathbf{r},\mathbf{\Omega},t) &= \kappa_a\Big[\frac{1}{4\pi}\sigma T(\mathbf{r},t)^4 - I(\mathbf{r},\mathbf{\Omega},t)\Big]\\ &+\kappa_s\Big[\frac{1}{4\pi}\int_{4\pi} I(\mathbf{r},\mathbf{\Omega},t)d\mathbf{\Omega} - I(\mathbf{r},\mathbf{\Omega},t)\Big]\\ &+S(\mathbf{r},\mathbf{\Omega},t) \end{aligned} \quad (1)$$

and

$$\begin{aligned}\rho C_p\frac{\partial T(\mathbf{r},t)}{\partial t}+\rho C_p\mathbf{v}.\nabla T &= \nabla.(k\nabla T)\\ &+\kappa_a\Big[\int_{4\pi} I(\mathbf{r},\mathbf{\Omega},t)d\mathbf{\Omega} - \sigma T(\mathbf{r},t)^4\Big]\end{aligned} \quad (2)$$

where $I(\mathbf{r},\mathbf{\Omega},t)$ is the radiation intensity at any spatial location $\mathbf{r}$ and time t in the direction $\mathbf{\Omega}$;$T(\mathbf{r},t)$ is the temperature; $\mathbf{v}$ is the velocity field of the fluid (material) stream; κ_a is the absorption cross-section; κ_s is the scattering cross-section; $S(\mathbf{r},\mathbf{\Omega},t)$ is the volumetric energy source; C_p is the specific heat; and σ is the Stefan–Boltzmann coefficient. These equations can also be written using total cross-section or attenuation coefficient $\beta=\kappa_a+\kappa_s$ as a parameter. The heat transport system where radiative transfer in participating media is significant, coupled simulation of Eqs. (1) and (2) is required. The velocity field in Eq. (2) is computed from fluid-momentum transport equations. The LBE solvers for fluid momentum and energy transport are now widely used for single phase and two phase calculations. Thus, LBE solver for Eq. (1) will help in building a single framework to solve fluid momentum, energy and radiation transport. The LBE solutions for fluid momentum and energy (convection–diffusion) transport has been discussed extensively in the past, so this work will use some of those widely accepted algorithms for the numerical examples. The LBE for the general form of RTE (Eq. (1)) will be discussed in the following subsection.

2.2. Lattice Boltzmann method for RTE

The standard Lattice Boltzmann Equations (LBE) model as derived previously in several studies is the discrete model in space, time and lattice velocities. This discrete model is used to evaluate distribution function for different lattice velocities. The LBE equations conserve the continuum mechanics equations for the macroscopic quantities such as mass, momentum and kinetic energy. A similar analogue for linear form of the general Boltzmann equation, which describes radiative transport process, has been developed with discrete angular directions for radiation field [17–19]. The distribution function or radiation intensity for each angular direction provides information for current and flux required in thermal energy balance calculations. The LBE for solving the radiation transport equations were formulated by Ma et al. [19], and were shown to conserve macroscopic energy flux using Chapman–Enskog expansion. They used a standard RTE with source term but without the scattering integral

$$\frac{\partial I(\mathbf{r},\mathbf{\Omega},t)}{\partial t}+\mathbf{\Omega}\cdot\nabla I(\mathbf{r},\mathbf{\Omega},t)+\sigma_a I(\mathbf{r},\mathbf{\Omega},t)=s_{\mathbf{\Omega}}(\mathbf{r},t). \quad (3)$$

and it was shown to have a corresponding LBE equivalent as

$$I_{\mathbf{\Omega}}(\mathbf{r}+\mathbf{\Omega}\Delta t,t+\Delta t)-I_{\mathbf{\Omega}}(\mathbf{r},t)=s_{\mathbf{\Omega}}(\mathbf{r},t)\Delta t-\sigma_a I_{\mathbf{\Omega}}(\mathbf{r},t)\Delta t. \quad (4)$$

The LBE for a general three dimensional system was modified by Bindra et al. [20] to incorporate weighted summations for scattering integrals. Using the taylor series expansion on these discrete LBEs, it can be shown that these equations are first order accurate in space and time. In this current work we will use this formulation to solve the RTE form stated in the previous section. The equivalent LBE for this RTE, Eq. (1), is

$$\begin{aligned}I_{\mathbf{\Omega}}(\mathbf{r}+\mathbf{\Omega}\Delta t,t+\Delta t)-I_{\mathbf{\Omega}}(\mathbf{r},t) &= \kappa_a\big[w_{\mathbf{\Omega}}T(\mathbf{r},t)^4-I_{\mathbf{\Omega}}(\mathbf{r},t)\big]\Delta t\\ &+\kappa_s\left[w_{\mathbf{\Omega}}\sum_{\mathbf{\Omega}'}I_{\mathbf{\Omega}'}(\mathbf{r},t)-I_{\mathbf{\Omega}}(\mathbf{r},t)\right]\Delta t\\ &+s_{\Omega}(\mathbf{r},t)\Delta t.\end{aligned} \quad (5)$$

For a two dimensional Cartesian LBE lattice, Eq. (5) reduces to

$$\begin{aligned}&I_i(x+v_{i,x}\Delta t,y+v_{i,y}\Delta t,t+\Delta t)=I_i(x,y,t)+v\Delta t\\ &\times\left(-\beta I_i(x,y,t)+\kappa_a w_i T(x,y,t)^4+\kappa_s w_i\sum_j I_j(x,y,t)+\kappa_a w_i s(x,y,t)\right),\end{aligned} \quad (6)$$

where $I_i(x,y,t)$ is the discrete angular neutron flux in the ith lattice direction at (x,y) location at time t. Lattice velocities $v_{i,x}$ and $v_{i,y}$ are chosen such that the distance traveled by particles in a discrete time interval Δt i.e., $v_{i,x}\Delta t$ and $v_{i,y}\Delta t$ equals to the distance between