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SOME CONDITIONS UNDER WHICH JORDAN DERIVATIONS ARE ZERO

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ABSTRACT. In the current article, we obtain the following results: Let $\mathcal{A}$ be an algebra and $\mathcal{P}$ be a semi-prime ideal of $\mathcal{A}$. Suppose that $d : \mathcal{A} \to \frac{\mathcal{A}}{\mathcal{P}}$ is a Jordan derivation such that $\dim\{d(a) \mid a \in \mathcal{A}\} \leq 1$. If $d(\mathcal{P}) = \{0\}$, then d is zero. As an application of this result, we prove that if $\mathcal{A}$ is an algebra such that $\bigcap_{\mathcal{P}\in\Sigma(\mathcal{A})} \mathcal{P} = \{0\}$, where $\Sigma(\mathcal{A})$ denotes the set of all semi-prime ideals of $\mathcal{A}$, and further each semi-prime ideal of $\mathcal{A}$ is of codimension 1, then $\mathcal{A}$ is commutative.

1. INTRODUCTION AND PRELIMINARIES

In this paper, $\mathcal{A}$ denotes an associative complex algebra. Recall that a proper ideal $\mathcal{I}$ of $\mathcal{A}$ is called prime if either $a \in \mathcal{I}$ or $b \in \mathcal{I}$, whenever $a\mathcal{A}b \subseteq \mathcal{I}$ for $a, b \in \mathcal{A}$. The set of all prime ideals of $\mathcal{A}$ is denoted by $\Pi_\rho(\mathcal{A})$. Moreover, a proper ideal $\mathcal{J}$ of $\mathcal{A}$ is said to be semi-prime if $a\mathcal{A}a \subseteq \mathcal{J}$ implies that $a \in \mathcal{J}$, for all $a \in \mathcal{A}$. We denote the set of all semi-prime ideals of $\mathcal{A}$ by $\Sigma(\mathcal{A})$. Obviously, every prime ideal is semi-prime, i.e. $\Pi_\rho(\mathcal{A}) \subseteq \Sigma(\mathcal{A})$. Furthermore, the set of all primitive ideals of $\mathcal{A}$ is denoted by $\Pi(\mathcal{A})$. Let $\mathcal{J}$ be an ideal of $\mathcal{A}$. The set $\frac{\mathcal{A}}{\mathcal{J}} = \{a + \mathcal{J} \mid a \in \mathcal{A}\}$ with the operations $(a + \mathcal{J})(b + \mathcal{J}) = ab + \mathcal{J}$, $(a + \mathcal{J}) + (b + \mathcal{J}) = (a + b) + \mathcal{J}$ and $\alpha(a + \mathcal{J}) = \alpha a + \mathcal{J}$ $(a, b \in \mathcal{A}$, $\alpha \in \mathbb{C})$ is called a quotient algebra (for more details see [2]).
A derivation on an algebra $\mathcal{A}$ is a linear mapping $d : \mathcal{A} \to \mathcal{A}$ which satisfies Leibnitz rule $d(ab) = d(a)b + ad(b)$ for all $a, b \in \mathcal{A}$. The linear mapping d is called a Jordan derivation if $d(a^2) = d(a)a + ad(a)$ for all $a \in \mathcal{A}$. It is clear that if d is a Jordan derivation on $\mathcal{A}$, then $d(ab + ba) = d(a)b + ad(b) + d(b)a + bd(a)$ for all $a, b \in \mathcal{A}$. In the current article, we study two historical subjects concerning derivations. The first subject regarding derivations of Banach algebras is their image and invariance of primitive ideals. Let us introduce a background of our study. In 1955, Singer and Wermer [11] obtained an outstanding result which started investigation into the range of derivations on Banach algebras. The result states that every continuous derivation on a commutative Banach algebra maps the algebra into its Jacobson radical. In the same paper, they wrote that it seems probable that the hypothesis of continuity in their theorem is superfluous. In 1988, M. P. Thomas [12] proved the conjecture. According to this result, every

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