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JTUSCI-350; No. of Pages 12
ARTICLE IN PRESS

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ScienceDirect

Journal of Taibah University for Science xxx (2017) xxx–xxx

Journal of Taibah University for Science

www.elsevier.com/locate/jtusci

Full Length Article

Investigation of the nonlinear equation of the circular sector oscillator by Akbari-Ganji's method

Hadi Mirgolbabaee *, Soheil Tahernejad Ledari, Navid Mohammad Zadeh, Davood Domiri Ganji

Department of Mechanical Engineering, Babol Noshirvani University of Technology, P.O. Box 484, Babol, Iran

Received 21 August 2016; received in revised form 26 October 2016; accepted 11 November 2016

Abstract

In this paper, a new and innovative semi-analytical method called Akbari-Ganji's method (AGM) has been applied to solve nonlinear equations of the semicircular oscillator. The major concern is to achieve an accurate solution that has an efficient approximation according to the Runge-Kutta numerical method. The results are presented for different values of parameters to demonstrate the applicability of this method. It was found that the proposed solution is very accurate and efficient for the discussed problem. It is worthwhile to mention that not only do convergence problems for solving nonlinear equations by using AGM appear small, but the results also demonstrate that the AGM could be applied to nonlinear problems with high nonlinearity.

Keywords: Akbari Ganji's Method (AGM); Angular frequency; Circular sector oscillation; Nonlinear equation; Numerical Method (Runge-Kutta 4th)

1. Introduction

Many oscillation systems have been used in physical phenomena and engineering. A wide range of problems in various engineering fields are explained in the form of nonlinear equations such as hydrodynamical machines, electrical engineering, heat transfer applications and civil engineering. Therefore, it is noteworthy to mention the influence of mathematical calculations in some recent research articles that are related to nonlinear differential equations arising in various science and engineering fields [1–4].

In this paper, attempts have been made to solve nonlinear equations of circular sector oscillation systems that could be mentioned as important problems in the free vibration field. The applicable analytical solution to this problem has been sought via different analytical methods [5]; however, it must be noted that in the procedure of utilizing analytical methods for solving nonlinear equations, it will be logical to apply methods that not only are accurate but also have simple solving processes, which will save time and reach the solution in a simple manner. According to the above explanation, this paper

* Corresponding author.
E-mail addresses: hadi.mirgolbabaee@gmail.com, h.mirgolbabaee@stu.nit.ac.ir (H. Mirgolbabaee).
Peer review under responsibility of Taibah University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.jtusci.2016.11.001
1658-3655

Please cite this article in press as: H. Mirgolbabaee, et al. Investigation of the nonlinear equation of the circular sector oscillator by Akbari-Ganji's method, J. Taibah Univ. Sci. (2017), http://dx.doi.org/10.1016/j.jtusci.2016.11.001

Nomenclature

AGM	Akbari-Ganji's Method
R	Semicircular radius
G	Gravity acceleration
A	Vibration amplitude
Greek letters	
α	Semicircular angle
$\theta(t)$	Angular displacement
$\dot{\theta}(t)$	Angular velocity

employs Akbari-Ganji's method (AGM) [6–9], which is a powerful and accurate method for solving nonlinear equations with respect to its simplicity through other semi-analytical methods.

Solving nonlinear equations is still a sophisticated problem. There are numerical methods to solve such equations, but during the procedure of solving, some important information including circular frequency will be lost. Therefore, analytical methods have emerged to solve nonlinear equations such as the Homotopy Perturbation Method [10,11], Variational Iteration Method [12–14], Adomian Decomposition [15,16], Homotopy Analysis Method (HAM) [17,18], Differential Transformation Method (DTM) [19,20], Energy Balance Method [21,22], Homotopy Analysis Transform Method (HATM) [23], q-Homotopy Analysis Transform Method (q-HAM) [24], Homotopy Analysis Sumudu Transform Method (HASTM) [25], and Exp-function Method [26–28].

The main purpose of AGM is to obtain an accurate solution with a simple algebraic calculation that is simpler than the other methods and that would be acceptable with minor errors compared with the numerical method. It is necessary to mention that a summary of the excellence of this method compared with the other approaches can be considered as follows: initial conditions are needed in accordance with the order of differential equations in the solution procedure, but when the number of initial conditions is less than the order of the differential equation, this approach can create additional new initial conditions regarding the own differential equation and its derivatives. Therefore, it is logical to mention that AGM is operational for miscellaneous nonlinear differential equations in comparison with the other methods.

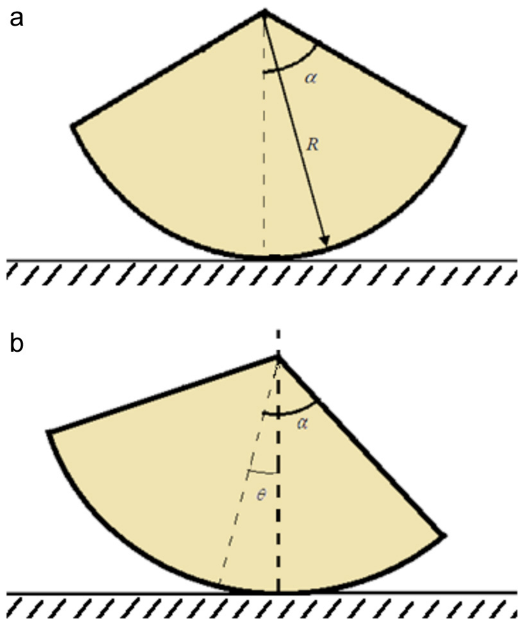

Fig. 1. Geometric parameters of the homogeneous solid circular sector body.

2. Swinging oscillation of a solid circular sector object

Consider a homogeneous solid circular sector object with angle α and radius R as shown in Fig. 1 that rolls in an oscillatory motion back and forth on a flat, stationary support, with no sliding effect [5]. Obviously, when α becomes radian, no oscillatory swinging motion will occur. It may be easily verified that the governing equation of the oscillation is as follows:

$$\left(\frac{3}{2}R^2 - \frac{4\sin(\alpha)}{3\alpha}R\cos(\theta)\right)\ddot{\theta} + R\left(\frac{2R\sin(\alpha)}{3\alpha}\sin(\theta)\right)\dot{\theta}^2 + \left(\frac{2\sin(\alpha)}{3\alpha}g\right)\sin(\theta) = 0 \tag{1}$$

$$\theta(0) = A,\ \dot{\theta}(0) = 0$$

By substitution of the relatively accurate approximations—$\sin(\theta) \approx \theta - \frac{\theta^3}{3!}$ and $\cos(\theta) \approx 1 - \frac{\theta^2}{2!}$—into Eq. (1), the governing equation would be in the following order:

$$\left(\frac{3}{2}R^2 - \frac{4\sin(\alpha)}{3\alpha}\left(1 - \frac{\theta^2}{2!}\right)\right)\ddot{\theta} + R\left(\frac{2R\sin(\alpha)}{3\alpha}\left(\theta - \frac{\theta^3}{3!}\right)\right)\dot{\theta}^2 + \left(\frac{2\sin(\alpha)}{3\alpha}g\right)\left(\theta - \frac{\theta^3}{3!}\right) = 0$$

$$\theta(0) = A,\ \dot{\theta}(0) = 0 \tag{2}$$