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Some new dynamic inequality on time scales in three variables

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Abstract

In this paper we obtain the estimates on some dynamic integral inequalities in three variables which can be used to study certain dynamic equations. We give some applications to convey the importance of our result.

Keywords: Explicit bounds; Integral inequality; Dynamic equations; Time scales

1. Introduction

The study of time scales was initiated in 1989 by Hilger [1] in his Ph.D. dissertation. Since then many authors have studied the dynamic inequalities on time scales. Some analytic inequalities on time scales in one and two variables is studied in [2–6] by various authors. The authors in [7–13] have obtained some interesting dynamic integral and iterated inequalities on time scales. Motivated by the results above in this paper we establish new explicit bounds on some dynamic inequalities in three variables which are useful in solving certain dynamic equations.

In what follows $\mathbb{R}$ denotes the set of real numbers, $I=[a, b]$ and $\mathbb{T}$ denotes arbitrary time scales. We say that $f : \mathbb{T} \to \mathbb{R}$ is rd-continuous provided f is continuous right dense point of $\mathbb{T}$ and has a finite left sided limit at each left dense point of $\mathbb{T}$ and will be denoted by C_{rd}. Let $\mathbb{T}_1$ and $\mathbb{T}_2$ be two time scales with at least two points and $\Omega = \mathbb{T}_1 \times \mathbb{T}_2$ and $H=\Omega \times I$. The basic information about time scales can be found in [14,15]. Now we give the Lemma given in [16] which is required in proving our result.

Lemma [16]: Let $u, a, f \in C'_{rd}(\mathbb{T}_1 \times \mathbb{T}_2, \mathbb{R}_+)$ and a is nondecreasing in each of the variables. If

$$u(x, y) \leq a(x, y) + \int_{x_0}^{x} \int_{y_0}^{y} f(s, t)u(s, t)\Delta t\Delta s, \tag{1.1}$$

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for $(x, y) \in \mathbb{T}_1 \times \mathbb{T}_2$ then

$$u(x, y) \leq a(x, y) e_{\int_{y_0}^{y} f(x,t)\Delta t}(x, x_0), \tag{1.2}$$

for $(x, y) \in \mathbb{T}_1 \times \mathbb{T}_2$.

2. Main results

Now we give our main result in the following theorem

Theorem 2.1. *Let $u, p_1, p_2, f \in C_{rd}(H, \mathbb{R}_+)$ and $c \geq 0$ be a constant. If*

$$u(x, y, z) \leq p_1(x, y, z) + p_2(x, y, z) \int_{x_0}^{x} \int_{y_0}^{y} \int_{a}^{b} f(s, \tau, q) u(s, \tau, q) \Delta q \Delta\tau \Delta s, \tag{2.1}$$

for $(x, y, z) \in H$, then

$$u(x, y, z) \leq p_1(x, y, z) + p_2(x, y, z) C(x, y) e_{Q(x,y,z)}(x, x_0), \tag{2.2}$$

where

$$Q(x, y, z) = \int_{y_0}^{y} \int_{a}^{b} f(s, \tau, q) p_2(s, \tau, q) \Delta q \Delta\tau \Delta s, \tag{2.3}$$

$$C(x, y) = \int_{x_0}^{x} \int_{y_0}^{y} \int_{a}^{b} f(s, \tau, q) p_1(s, \tau, q) \Delta q \Delta\tau \Delta s. \tag{2.4}$$

Proof. Now let

$$M(s, t) = \int_{a}^{b} f(s, \tau, q) p_2(s, \tau, q) \Delta q. \tag{2.5}$$

Then (2.1) becomes

$$u(x, y, z) \leq p_1(x, y, z) + p_2(x, y, z) \int_{x_0}^{x} \int_{y_0}^{y} M(s, \tau) \Delta\tau \Delta s. \tag{2.6}$$

Now put

$$W(x, y) = \int_{x_0}^{x} \int_{y_0}^{y} M(s, \tau) \Delta\tau \Delta s. \tag{2.7}$$

Then $W(x, y_0) = W(x_0, y) = 0$ and

$$u(x, y, z) \leq p_1(x, y, z) + p_2(x, y, z) W(x, y). \tag{2.8}$$

From (2.7), (2.5), (2.8) we have

$$\begin{aligned} W^{\Delta_1 \Delta_2}(x, y) = M(x, y) &= \int_{a}^{b} f(x, y, q) u(x, y, q) \Delta q \leq \int_{a}^{b} f(x, y, q)[p_1(x, y, z) + p_2(x, y, z) W(x, y)] \Delta q \\ &= W(x, y) \int_{a}^{b} f(x, y, q) u(x, y, q) \Delta q + \int_{a}^{b} f(x, y, q) p_1(x, y, q) \Delta q \\ &= \int_{a}^{b} f(x, y, q) p_2(x, y, q) \Delta q + \int_{a}^{b} f(x, y, q) p_1(x, y, q) \Delta q. \end{aligned} \tag{2.9}$$

Now from (2.9) above we have by taking delta integral

$$W^{\Delta_1}(x, y) \leq \int_{y_0}^{y} \int_{a}^{b} W(x, \tau) f(x, \tau, q) p_2(x, \tau, q) \Delta q \Delta\tau + \int_{y_0}^{y} \int_{a}^{b} f(x, \tau, q) p_1(x, \tau, q) \Delta q \Delta\tau. \tag{2.10}$$

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