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# Modification of the boundary element method for computation of three-dimensional fields of strain–stress state of cavities with cracks

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ABSTRACT

In order to solve the three-dimensional problem of crack initiation in a cavity and propagation in an elastic medium under the effect of pumping a viscous liquid inside the cavity, a modification of the boundary element method is used for determining the strain–stress state in vicinity of the cavity with the crack connected to it, loaded with the pumped-in viscous liquid. The foundation of the method is equations of elastic equilibrium written in the form of boundary integral equations. The conventional boundary element method cannot be used for this problem because the boundary integral equation for displacements degenerates at the crack. In the known dual boundary element method, with the purpose of eliminating this drawback of the conventional method, an additional boundary integral relation at the discontinuous part of the boundary is constructed. Therewith, discontinuing elements are introduced, which allow to approximate integral relations and regularize and calculate singular integrals. The goal of this study is to modify the dual boundary element method for three-dimensional model of crack propagating from cavity under the effect of pumped-in viscous liquid in an elastic unbounded medium. One of the problems is a fully three-dimensional modeling of the hydraulic crack process starting from well, for the solution of which the developed profit-proved modification of the dual method was successfully employed. The research results are presented thereof.



## 1. Introduction

Among a great number of papers dedicated to modeling of the formation hydraulic fracturing process, a special place is held by papers dedicated to fully three-dimensional models [1–9]. The important feature of such models is the ability to describe out-of-plane propagation, or, in other words, spatial reorientation [2,4,9]. They describe not only normal fracture, but also relative shear displacements of crack edges and allow modeling crack propagation of shear and rupture types [2,10]. A typical case when such models are needed is: the initial crack has a non-preferred orientation, and tortuous final crack may result. Then the restriction of the crack width near the wellbore will lead to plugging it with proppant agent [11,12]. In this paper, we focus on the simulating exclusively full 3D evolution of hydraulic fractures. The main sub-model of the hydrofracture model is the problem of determination of the strain–stressed state in the vicinity of the cavity (well) surface and the

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**Nomenclature**

| | |
|---|---|
| FEM | finite element method |
| BEM | boundary element method |
| DDT | displacement discontinuity technique |
| DBEM | dual boundary element method |
| DBIE | displacement boundary integral equation |
| TBIE | traction boundary integral equation |
| LEFM | linear elastic fracture mechanics |
| SIFs | stress intensity factors |

crack propagating from it under the effect of the liquid pumped into them. In all papers cited herein, as well as in this paper itself, elastic equilibrium equations are used for determination of the stress–strain state.

As the method for solution of the elasticity sub-problem in the studies dedicated to modeling of the 3D evolution of hydraulic fractures, essentially two main options are used: finite element method (FEM) [2] or boundary element method (BEM) [4], which prototype could be the so-called "direct method" of linear elasticity based on Somigliana's identity [13]. In the first case, it is supposed to discretize the 3D partial differential equations, and then a large volume of the reservoir in the vicinity of the wellbore and hydraulic fracture needs to be discretized. This is very expensive computationally. Because of that, BEM-like approaches are more frequently used for solving the elasticity sub-problem, which automatically satisfy the condition of no displacement at infinity, and no approximation of the medium volume up to a remote boundary is needed [4,9]. However, not all BEM-like approaches can be used for the problems of crack propagation from cavity.

Conventional BEM [13] may be employed for solution of the initiation problem, if it only deals with a cavity confined by a surface $S^*$, and there are no cracks [14–16]. However, using the conventional BEM to collocate on coincident points on the opposing crack surfaces gives rise to a singular system of algebraic equations. The equations for a point located on one of the surfaces of the crack are identical to those equations for the point with the same coordinates, but on the opposite surface [17,18].

Many methods have been developed to overcome this difficulty. The crack Green's function method [19] is applied to solve problems with a dominant crack of such a regular shape that free-space Green's functions, which satisfies the traction-free boundary condition on the crack surface, is obtainable.

The multiple-zone method [20] introduces artificial boundaries in the intact area to connect cracks and the boundary and thus divides the domain into such zones that no cracks could appear in the interior of each zone. The setback of the multiple-zone method is that the introduction of artificial boundaries is not unique, and thus cannot be easily implemented into an automatic procedure. At the same time, solving the problem of crack propagation requires rebuilding artificial boundaries at each step of the crack growth. In addition, the method generates a larger system of algebraic equations than is strictly required. Despite these setbacks, the multiple-zone method has been the most widely used technique for elastostatics. In [21,22], multizone BEM was used for solving a 3D problem of crack initiation in a cased well. The drawback of the technique of usage of multizone BEM for solving the problem of crack propagation from cavity is the necessity of rearrangement of an artificial boundary at each step of crack growth.

The displacement discontinuity method [23], where the unknown functions are the displacement differences between the crack surfaces, can be used directly. The use of derivatives in the formulation introduces higher order singularities into the boundary integrals [24]. In one of the first papers on simulation of fully three-dimensional fracture propagation [3], for solving equations of elastic equilibrium, the displacement discontinuity technique (DDT) is used. The method is based on analytical solution of the infinite displacement plane problem, in which displacements are discontinued within a finite interval. The drawback of DDT is that it cannot be used for problems of cracks' propagation from cavity. It can only be employed for isolated cracks.

The most suitable method for solving the elasticity problem in the proposed three-dimensional model of crack propagation from the cavity is the dual boundary element method (DBEM) [25–27]. The use of dual integral equations in solving crack problems was first introduced by Bueckner [26]. Watson [28,29] presented the normal derivative of the displacement boundary integral equation for the development of Hermite cubic element where the number of the unknowns is larger than the number of equations. For the case of a degenerated boundary, the dual integral representation has been proposed for solving crack problems by Hong and Chen [25,27]. They introduced the idea of dual boundary integral equation, in which a combination of the standard boundary integral equation and its derivative can be used to provide independent equations with a view to overcome the degeneracy problem. Hong and Chen presented the theoretical bases of the dual integral equations, showing how the displacement boundary integral equation can be differentiated and how Hooke's law could be applied to derive the traction boundary integral equation. Based on the dual integral representation, Hong and Chen developed the program related to the dual boundary element method for solving the crack and potential flow problems. They point out that the definition of the dual boundary integral equations is quite different from the dual integral equations provided by Bueckner [26], which, indeed, come from the same equation but with regard to different collocation points in solving the crack problems in elastodynamics. Portela et al. [30] implemented the combined use of the displacement boundary