Engineering Fracture Mechanics 160 (2016) 238–247

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Comparison of toughness propagation criteria for blade-like and pseudo-3D hydraulic fractures

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ARTICLE INFO

Article history:
Received 2 November 2015
Received in revised form 6 April 2016
Accepted 11 April 2016
Available online 20 April 2016

Keywords:
Hydraulic fracturing
PKN model
Pseudo-3D model
Non-local elasticity

ABSTRACT

The goal of this study is to compare and evaluate the accuracy of different approaches to incorporating the effect of lateral fracture toughness into reduced models for blade-like and pseudo-3D hydraulic fractures. The following three methods are used for the comparison: (i) a classical model with a plane strain (or local) elasticity assumption and a pressure boundary condition calculated based on energetic considerations, (ii) a classical model with local elasticity and pressure boundary condition originating from "stitching" a radial fracture tip to the rest of the fracture, and (iii) a novel model with non-local elasticity and a boundary condition at the tip that is consistent with the linear elastic fracture mechanics propagation criterion. Predictions of all three approaches are compared to a reference solution calculated using a fully planar hydraulic fracturing simulator. The results indicate that the reduced model with non-local elasticity is able to provide an accurate approximation for a wide range of fracture toughness values. The models that feature the local elasticity assumption are able to provide reasonably accurate results for moderate values of fracture toughness, while they become less accurate for blade-like geometries and significantly less accurate (and in some cases unstable) for the pseudo-3D geometry for large values of the fracture toughness.

1. Introduction

The ability to model a propagating hydraulic fracture is an essential tool for designing a hydraulic fracture treatments. Multiple approaches can be employed. For instance, the fracture geometry can be straight or curved in plane strain elastic conditions [1,2], penny-shaped (radial) [3], planar [4,5], or a system of fractures [6,7] can be analyzed. This study focuses solely on the propagation of planar vertical hydraulic fractures. In particular, two fracture geometries are considered, namely the Perkins–Kern–Nordgren (PKN) [8,9] (or blade-like) fracture geometry and pseudo-3D (P3D) [10–13] fracture geometry with symmetric stress barriers. Both PKN and P3D models are reduced models, since they use a series of approximations that reduce the complexity (and dimensionality) of the problem, making the resulting method computationally efficient. In the original formulations, however, both the PKN and P3D models lack a toughness propagation criterion in the lateral direction, which causes significant discrepancies in situations when fracture toughness is dominant. Several improvements have been made to account for the effect of fracture toughness. This paper aims to summarize and evaluate the accuracy of the available approaches for both PKN and P3D fracture geometries.

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http://dx.doi.org/10.1016/j.engfracmech.2016.04.023
0013-7944/

Nomenclature

x, y, z	spatial coordinates
t	time
H	height of the reservoir layer
$h(x,t)$	fracture height
$l(t)$	fracture half-length
$\Delta\sigma$	magnitude of stress barriers
w	fracture width
$\bar{w}$	effective width
$E' = E/(1-\nu^2)$	plane strain Young's modulus
p	fluid pressure
K_{Ic}	fracture toughness
$\mu' = 12\mu$	scaled fluid viscosity
Q_0	inlet flux
$\bar{q}_x$	vertically-averaged fluid flux
U_{ps}	elastic energy density in plane strain
G_c	fracture energy per unit area

The PKN model considers a vertical planar hydraulic fracture that propagates laterally, while the fracture height is constant throughout the fracture. This situation occurs when a reservoir layer is surrounded by two impenetrable layers that arrest the fracture propagation in the vertical direction. Due to the elongated shape of the fracture, plane strain elastic conditions prevail in each vertical cross-section (away from fracture tip), and the fluid flow is predominantly horizontal. The latter implies that the pressure is constant in each vertical cross-section, which, together with the plane strain (or so-called local) elasticity assumption, leads to the conclusion that each fracture cross-section has an elliptical shape. The knowledge of the fracture width profile in the vertical direction permits one to formulate the problem in terms of a vertically-integrated lubrication equation, which reduces the dimension of the problem and makes the model computationally efficient. Clearly, the model assumptions are violated near the fracture tip, since the plane strain elasticity assumption does not hold near the fracture tip region. In situations when a PKN fracture propagates in the viscous regime (viscous dissipation dominates), the fracture tip region does not have a significant influence on the solution since viscous dissipation is distributed throughout the fracture. However, when a PKN fracture propagates in the toughness regime (fracture energy dissipation dominates), the fracture tip region has a substantial impact since the fracture energy is dissipated at the fracture tip. For this reason, the original formulation of the PKN model is not able to capture effect of fracture toughness accurately. One correction has been proposed by Nolte [14], in which a pressure boundary condition at the tip is used to capture the effect of fracture toughness. The value of the latter pressure is taken from the solution for a uniformly pressurized penny-shaped fracture, whose diameter is equal to the fracture height. In this approach half of the radial fracture that resembles the fracture tip is "stitched" to the rest of the fracture. Another approach has been recently introduced in [15], in which a different pressure boundary condition has been proposed. The approach utilizes energy considerations, for which the elastic energy release rate (calculated assuming plane strain elasticity in each vertical cross-section) is equated to the fracture energy required to break the rock ahead of the fracture tip. The difference between two proposed values for the pressure boundary condition is approximately 10%. A qualitatively different approach to capture the effect of fracture toughness has been suggested in [16], where the local elasticity assumption has been replaced by non-local elasticity (with a suitable propagation criterion that is consistent with linear elastic fracture mechanics), which remains valid even near the fracture tip. The results in [16] are presented for a pseudo-3D fracture geometry and demonstrate an excellent agreement with the reference solution even for large values of fracture toughness. Since the results in [16] do not consider the PKN fracture geometry, this study aims to describe an analogous formulation with non-local elasticity for the PKN fracture geometry, and to compare its performance to the existing corrections for the effect of fracture toughness [14,15] and a reference solution. It should be noted here that the analysis of the non-local elasticity equation for the PKN fracture was first done in [17], while no numerical results for a PKN model with non-local elasticity and coupled fluid dynamics were presented.

The classical pseudo-3D (P3D) model with symmetric stress barriers [13] is an extension of the PKN model, where a vertical fracture growth is allowed. Similar to the PKN fracture, the P3D model assumes a uniform pressure in each vertical cross-section and uses plane strain (or local) elasticity to obtain a solution for the vertical fracture width profile. The primary difference comes from the presence of stress barriers, which introduce an additional compressive stress in the layers above and below the reservoir layer. Note that the values of the elastic constants are assumed to be identical in all layers. The stress barriers change the elliptical shape of the fracture width cross-section (for a PKN fracture) to a more complicated shape, which is given by an analytical function. As for the PKN model, the governing equation for a P3D fracture is a vertically-averaged lubrication equation, in which case the computations are reduced to solving a one-dimensional problem making the P3D model extremely computationally efficient. Since the plane strain elasticity assumption becomes invalid near the fracture tip (as for a PKN fracture), a P3D model is unable to accurately capture the effect of fracture toughness (as the