Ultrasonics - Sonochemistry xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Ultrasonics - Sonochemistry

journal homepage: www.elsevier.com/locate/ultson

Chaotic oscillations of gas bubbles under dual-frequency acoustic excitation

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ARTICLE INFO

Keywords:
Chaos
Bifurcation
Dual frequency
Gas bubbles

ABSTRACT

Chaotic oscillation of bubbles in liquids reduces the efficiency of the sonochemical system and should be suppressed in the practical applications. In the present paper, a chaos control method based on the dual-frequency approach is numerically investigated and is proved to be an effective method even for cases with intensive energy input. It was found that the chaos could be successfully suppressed by the application of dual-frequency approach in a wide range of parameter zone (even with high acoustic pressure amplitude). Furthermore, influences of power allocation between two waves on the chaos control are quantitatively discussed with clear descriptions of the routes from stable oscillations to chaos.

1. Introduction

Currently, acoustic cavitation is being widely employed in the sonochemistry including the promotions of the chemical reactions [1,2], the chemical synthesis [3], the surface cleaning [4] and nanostructuring [5]. In many applications of the cavitation effects, the chaotic oscillations of bubbles should be avoided because these bubbles will not oscillate following a controllable route, leading to the difficulties in the design of sonochemical reactors. Therefore, in order to avoid some undesirable results (e.g. efficiency reduction of the sonochemical reactor), control of the chaotic oscillations of the bubbles is an essential topic for the cavitation-enhanced sonochemical effects.

Among the chaos control strategies, dual-frequency technique is one of important chaos control method for the nonlinear system with following characteristics. Firstly, it is convenient for the implementation. In the dual-frequency chaos control method, the existing system does not need much modification except adding an extra acoustic wave into the system for the purpose of the suppress of the chaos. Secondly, the dual-frequency approach has been widely proved to be a highly effective method for many systems. In the literature, the nonlinear systems with successful chaos control include the electrical circuits, the duffing oscillator, the duffing-van der Pol oscillator, and the Frenkel-Kontorova chains. For reviews of dual-frequency chaos control technique, readers are referred to Nayfeh and Mook [6], Fradkow *et al.* [7] and Chacón [8]. Thirdly, the dual-frequency approach is suitable for the bubble oscillator. For the oscillating bubbles in certain types of applications, many parameters of the system (e.g. the viscosity, the surface tension, and the speed of sound of surrounding liquids) are all fixed by the types of applications. Hence, as an external field, the dual-frequency approach could be independent of the types of applications, being a general method for many emerging fields. In the literature, the dual-frequency approach has been proved to be effective for promoting cavitation bubble effects [9–21]. Furthermore, the bubble dynamics has been actively involved into the design of the sonochemical reactors [15]. In the literature, many physical effects induced by the dual-frequency excitation have been intensively studied, including the mass transfer [16,17], the phase diagrams [14], the acoustical scattering [18], the secondary Bjerknes force between bubbles [19], the combination and simultaneous resonances [20], and the surface instability [21]. Specifically, Zhang [16] found that the dual-frequency approach could significantly accelerate the bubble growth through mass transfer process, leading to the generation of large bubbles (as shown in figure 1 of ref. [16]). Zhang et al. [20] further revealed two unique features (combination resonance and simultaneous resonance respectively) of bubble oscillations under dual-frequency acoustic excitation. They also gave a quantitative evaluation of the combination resonances in terms of power in the frequency-response curve together with the traditional main bands, harmonics and subharmonics. Furthermore, dual-frequency technique could be also applied to the modification of bubble distributions in the practical applications through changing the sign of forces between bubbles [19]. For the nonlinearity (e.g. bifurcation), Behnia *et al.* [12] briefly studied the response of a spherical bubble to a special case of dual-frequency technique (with equal amplitudes of two acoustic sources). However, the ability of the dual-frequency approach for supressing the chaos has not been studied in great detail for the general purpose (e.g. within a large parameter zone).

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http://dx.doi.org/10.1016/j.ultsonch.2017.03.058
Received 16 November 2016; Received in revised form 17 February 2017; Accepted 27 March 2017

Please cite this article as: Zhang, Y., Ultrasonics - Sonochemistry (2017), http://dx.doi.org/10.1016/j.ultsonch.2017.03.058

In the present paper, the dual-frequency chaos control method is numerically investigated within a wide range of parameter zone (e.g. the power allocation between two acoustic waves and acoustic amplitude). The sections of the present paper are organized as follows. Section 2 introduces the basic equations and numerical methods for studying the chaotic oscillations of bubbles. Section 3 discusses the methods for chaos identification. Section 4 proves the effectiveness of the dual-frequency chaos control method through several examples. Section 5 further reveals the examples of the route to chaos in the dual-frequency approach. Section 6 concludes the main findings of the present paper with remarks.

2. Equations and methods

In this section, the equations and method employed for the prediction of the chaotic oscillations of gas bubbles in liquids will be introduced. The equation of bubble interface motion adopted here is the Keller-Miksis equation [22],

$$\left(1-\frac{\dot{R}}{c_l}\right)R\ddot{R}+\frac{3}{2}\left(1-\frac{\dot{R}}{3c_l}\right)\dot{R}^2=\left(1+\frac{\dot{R}}{c_l}\right)\frac{p_{ext}(R,t)-p_s(t)}{\rho_l}+\frac{R}{\rho_l c_l}\frac{d[p_{ext}(R,t)-p_s(t)]}{dt}, \quad (1)$$

where

$$p_{ext}(R,t)=\left(P_0+\frac{2\sigma}{R_0}\right)(R_0/R)^{3\kappa}-\frac{4\mu_l}{R}\dot{R}-\frac{2\sigma}{R}, \quad (2)$$

$$p_s(t)=P_0[1+\varepsilon_1\cos(\omega_1 t)+\varepsilon_2\cos(\omega_2 t+\varphi)]. \quad (3)$$

Here R is the instantaneous bubble radius; the overdot denotes the time derivative; c_l is the speed of sound in the liquid; ρ_l is the density of the liquid; t is the time; P_0 is the ambient pressure; σ is the surface tension coefficient; R_0 is the equilibrium bubble radius; κ is the polytropic exponent; μ_l is the viscosity of the liquid; ε_1 and ε_2 are the non-dimensional amplitudes of the external acoustic excitation; ω_1 and ω_2 are the angular frequencies of the two external acoustic waves (assuming that $\omega_1 < \omega_2$ for convenience); φ is the relative phase between the two external acoustic waves. For convenience, we set $\varphi = 0$ here. Also, the energy dissipation through the thermal effects was ignored [23,24]. Eqs. (1)–(3) is an ordinary differential equation and could be directly solved by using the fourth order Runge-Kutta method.

For convenience of discussions, the non-dimensional parameters are employed as listed as below

$$X_{\max}=(R_{\max}-R_0)/R_0; \quad (4)$$

$$\tilde{\omega}_1=\frac{\omega_1}{\omega_0};\ \tilde{\omega}_2=\frac{\omega_2}{\omega_0} \quad (5)$$

$$\frac{P_e}{P_0}; \quad (6)$$

$$N=\frac{\varepsilon_2}{\varepsilon_1}; \quad (7)$$

with

$$\omega_0^2=\frac{1}{\rho_l R_0^2}\left[3\kappa\left(P_0+\frac{2\sigma}{R_0}\right)-\frac{2\sigma}{R_0}\right], \quad (8)$$

$$P_e=P_0\varepsilon_1^2(1+N^2). \quad (9)$$

Here $R_{\max}$ is the local maximum of the bubble radius during oscillations; ω_0 is the linear natural frequency of the oscillating bubbles; P_e is the overall acoustic amplitude of the dual-frequency excitation. The following constants are employed in calculations: $\rho_l = 998.20$ kg/m^3; $\mu_l = 1.0$ mPa·s; $\sigma = 0.0728$ N/m; $P_0 = 1.013 \times 10^5$ Pa; $R_0 = 10$ μm; $c_l = 1486$ m/s; $\kappa = 1.33$.

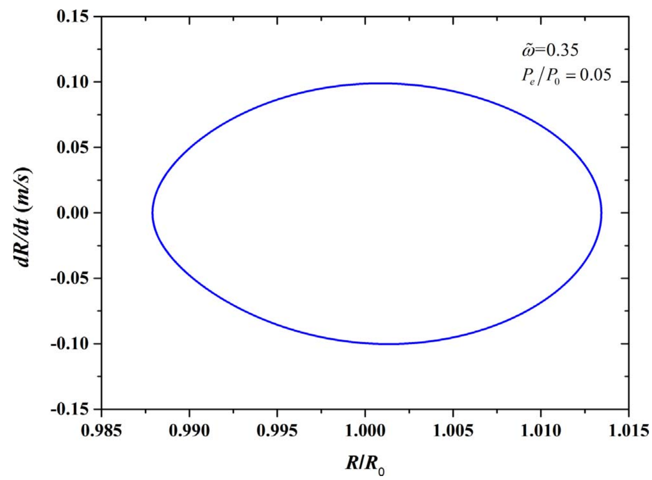

Fig. 1. Variations of the bubble interface motion speed (dR/dt) versus the non-dimensional bubble radius (R/R_0) under single-frequency excitation without chaos. $\tilde{\omega} = 0.35$. $P_e/P_0 = 0.05$.

3. Method for chaos identification

In this section, the method employed in the present paper for the identification of chaos will be introduced. Firstly, cases with single-frequency excitation will be investigated. For single-frequency excitation, the frequency of acoustic wave is represented simply by ω. According to Eqs. (5) and (8), the frequency could be non-dimensionalized as $\tilde{\omega} = \omega/\omega_0$. The overall acoustic amplitude of the single-frequency excitation is still denoted as P_e. Fig. 1 shows the variations of the bubble interface motion speed (dR/dt) versus the non-dimensional bubble radius (R/R_0) under single-frequency excitation without chaos. In Fig. 1, acoustic wave with a small amplitude ($P_e/P_0 = 0.05$) is employed. As shown in Fig. 1, the bubble oscillates periodically with a fixed pattern. However, when the amplitude increases, the bubble behaviour will be quite different. Fig. 2 shows the oscillations of the same bubble in Fig. 1 but with larger amplitude ($P_e/P_0 = 0.85$). As shown

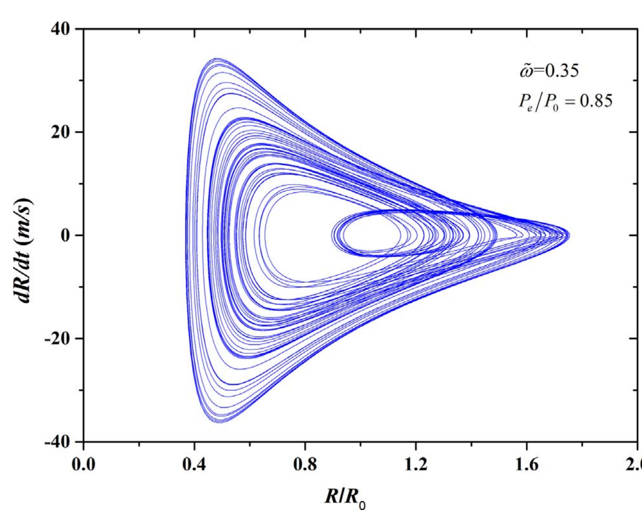

Fig. 2. Variations of the bubble interface motion speed (dR/dt) versus the non-dimensional bubble radius (R/R_0) under single-frequency excitation with chaos. The bubble oscillates chaotically. $\tilde{\omega} = 0.35$. $P_e/P_0 = 0.85$.