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Nonlinear fracture mechanics analysis of nano-scale piezoelectric double cantilever beam specimens with surface effect

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ABSTRACT

A fracture mechanics analysis model for the piezoelectric double cantilever beam (DCB) fracture specimens is developed. The model incorporates residual surface stress, surface elasticity and surface piezoelectricity and considers the finite deformation theory. Based on Timoshenko beam theory, the governing equation is derived and solved numerical. Consideration of geometrically nonlinear deformation will significantly decrease the prediction of energy release rate. The effects of residual surface stress and surface elasticity on the energy release rate is more significant for a thinner beam. The influence of surface residual stress on the energy release rate depends on the length to thickness ratio of the DCB. For open-circuit boundary condition, the applied voltage and surface piezoelectricity can affect the energy release rate, and the effect of surface piezoelectricity is more obvious for a thinner beam. However, for short-circuit boundary condition the surface piezoelectricity cannot affect the energy release rate.

1. Introduction

Double cantilever beam (DCB) specimen (as shown in Fig. 1(a)) is an attractive test configuration for both experimental and theoretical studies on crack propagation and arrest. DCB specimen has been widely used in determination of energy release rate and mode I fracture toughness of various materials, such as homogeneous, composite and adhesively bonded materials (Saenz et al., 2012; Guo et al., 2006; Zhang et al., 2013; Corrado and Paggi, 2015).

Nanomaterials have been widely used in MEMS/NEMS. The fracture characterizations of nanomaterials are different from that of macro materials (Kitamura et al., 2003, 2004). Understanding the unique fracture behaviors of nanomaterials is essential and necessary. Due to the sizes of structures reduce to micro/nano-scale, the surface area to bulk ratios of nanostructures are huge. At this situation, the influence of surface stress on the mechanical behaviors of nanostructures is significant (Fang et al., 2015). In fact, the effect of surface stress on the fracture characterizations of nano-scale materials is significant (Fang et al., 2014; Fu et al., 2010; Wang and Wang, 2013; Nan and Wang, 2013; Gao et al., 2014; Wang et al., 2013; Li et al., 2006). For example, Fu et al. (2010) analyzed the influence of surface stress on the stress distribution of mode-I crack, by using finite element method (FEM). The effect of residual surface stress on the fracture of DCB specimen is investigated for small deformation of the specimen and without consideration of surface elasticity (Wang and Wang, 2013).

Mixed-mode bending (MMB) tests show that the errors in energy release rate calculated by using linear theory are larger than 30%, in some cases, while the errors of the redesigned mixed-mode bending apparatus with considering geometric nonlinearity are less than 3% (Reeder and Crews, 1991). In tests of DCB, Devitt et al. (1980) and Williams (1987) also found that the effect of geometric nonlinearity on the mode I fracture toughness of composite materials is suffice for long cracks. However, in the models developed by Devitt et al. (1980) and Williams (1987), the surface effect is not considered. This will result in inaccurate predictions of the fracture toughness of nanomaterials. Moreover, the effects of shear deformation and the un-cracked end bulk of the DCB are not taken into account.

On the other hand, piezoelectric nanomaterials have extensive applications in nanodevices, such as nanoresonators (André, 2010) and nanogenerators (Yang et al., 2009). In order to fulfill the potential applications of those nanodevices, it is essential to well

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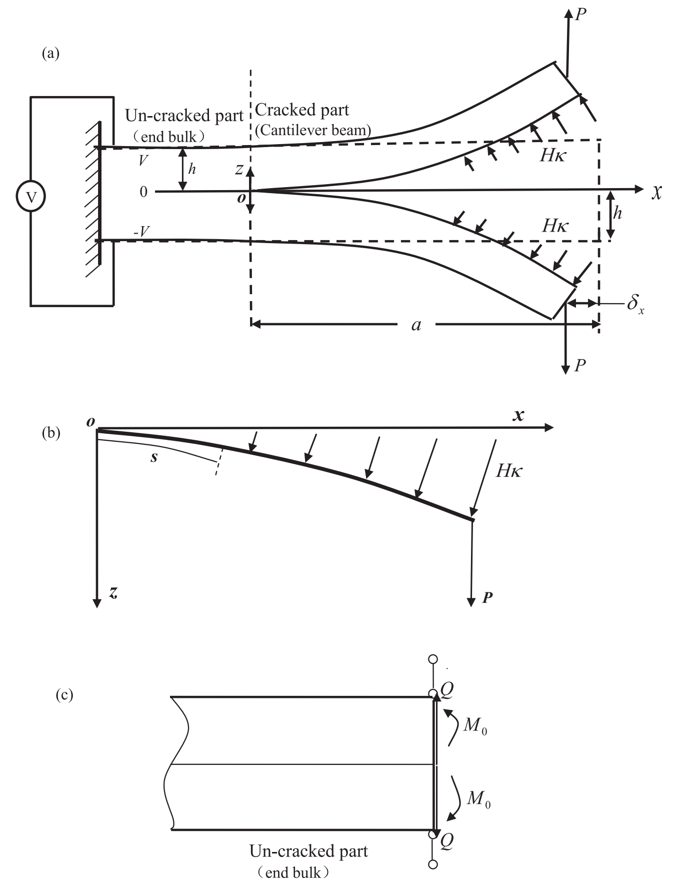

Fig. 1. (a) The double cantilever beam (DCB) specimen; (b) Cantilever beam model for large deflection; (c) End part.

understand the failure behavior of the piezoelectric nanomaterials. The failure behavior of the piezoelectric macromaterials has been widely studied (Zhang and Xie, 2012, 2013). However, studies on failure behavior of the piezoelectric nanomaterials are few. For example, Nan and Wang (2013) found that the influence of surface effect on the energy release rate, stress and electric field intensity factors for piezoelectric nanomaterials is significant. Due to the facts that DCB specimen is an attractive test configuration for studies on fracture toughness and commonly subjected to large deformation (Devitt et al., 1980; Williams, 1987), and the effect of surface stress on the fracture of nanomaterials is significant, this paper investigates the fracture toughness of nano-scale piezoelectric DCB specimen with simultaneous consideration of large deformation, shear deformation, un-cracked end bulk, residual surface stress, surface elasticity and surface piezoelectricity. Results show that the effect of large deformation on the energy release rate becomes more significant for a large end point force.

2. Problem formulation

As shown in Fig. 1(a), a piezoelectric DCB specimen with the length of cracked part a, width b, thickness h and a point force (P) applied to the free end of the beam, is studied. Similar to Refs (Huang and Yu, 2006; Yan and Jiang, 2011; Zhang et al., 2008; Shen and Hu, 2010), the “bulk + surface” model is used to model the DCB. Based on Timoshenko beam theory, the constitutive relations of bulk can be expressed as

$$\sigma_{xx} = -c_{11} z \frac{d\varphi}{ds} - e_{31} E_z, \quad \sigma_{xz} = G(\theta - \varphi) - e_{15} E_x \tag{1}$$

$$D_z = -e_{31} z \frac{d\varphi}{ds} + k_{33} E_z, \quad D_x = e_{15} \gamma_{xz} + k_{11} E_x \tag{2}$$

where θ and φ are the slope of the deformed beam and the rotation of the cross section, respectively. c_{11} and G are the Young's module and shear module of bulk; D_z and D_x are the electric displacements; k_{11} and k_{33} are the dielectric constants of the bulk; e_{31} and e_{15} are piezoelectric constants of the bulk.

For the surface layer, the stress and electric displacement of surface layer can be expressed as (Huang and Yu, 2006)

$$\tau_x = \tau_0 + c_{11}^s \varepsilon_{xx} - e_{31}^s E_z \tag{3}$$

$$D_x^s = D_x^0 + e_{15}^s \gamma_{xz} + k_{11}^s E_x \tag{4}$$

where c_{11}^s is Young's module of surface. τ_0 is residual surface stress. D_x^0 is residual electric displacements. e_{15}^s and e_{31}^s is piezoelectric constants of the surface.

The electric-field components can be expressed as

$$E_x = -\Psi_{,x}, \quad E_z = -\Psi_{,z} \tag{5}$$

where Ψ is the electric potential. It has been shown that the electric potential is almost a constant along the nanobeam span (the x-axis) (Yan and Jiang, 2011). Naturally, it is known that $E_x << E_z$, so that $D_x << D_z$.Therefore, the electric displacement D_x can be neglected. In the absence of body electric charges, the equilibrium equation is $D_{z,z} = 0$. Using Eq. (5) and the boundary conditions $\Psi(-h/2) = 0$ and $\Psi(h/2) = V$ for the upper beam, one obtains

$$\Psi = -\left(\frac{e_{31}}{k_{33}} \frac{\partial\varphi}{\partial s}\right)\left(\frac{z^2 - (h/2)^2}{2}\right) + V\frac{z}{h} + \frac{V}{2} \tag{6}$$

$$E_z = -\frac{\partial\Psi}{\partial z} = z\left(\frac{e_{31}}{k_{33}} \frac{\partial\varphi}{\partial s}\right) - \frac{V}{h} \tag{7}$$

Therefore, the stress of bulk and surface can be expressed as

$$\sigma_{xx} = -z\frac{\partial\varphi}{\partial s}\left(c_{11} + \frac{e_{31}^2}{k_{33}}\right) + \frac{e_{31} V}{h} \tag{8}$$

$$\tau_x = \tau_0 - z\left(c_{11}^s + \frac{e_{31}^s e_{31}}{k_{33}}\right)\frac{\partial\varphi}{\partial s} + \frac{e_{31}^s V}{h} \tag{9}$$

The moment of the cross section is

$$M = \int_A \sigma_{xx} z dA + \int_S \sigma_s z dS = -EI_{eff} \frac{\partial\varphi}{\partial s} \tag{10}$$

where
$EI_{eff} = (c_{11} + e_{31}^2/k_{33}) bh^3/12 + (c_{11}^s + e_{31}^s e_{31}/k_{33})(h^3/6 + bh^2/2)$.
The force induced by electrical of cross section is

$$T = \int_A \sigma_x dA + \int_s \tau_x ds = e_{31} Vb + 2bVe_{31}^s/h \tag{11}$$

By using the stress-strain relations, we obtained

$$-EI_{eff} \partial\varphi/\partial s = M \;\text{ and }\; G^s A(\theta - \varphi) = Q \tag{12}$$