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Path-independent integral in fracture mechanics of quasicrystals

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ABSTRACT

Path-independent integrals are successfully utilized for accurate evaluation of fracture parameters in crystalic materials, where atomic arrangement is periodic. In quasicrystals (QC) the atomic arrangement is quasiperiodic in one-, two- or three-directions. The 2-d elastic problem for quasicrystal is described by coupled governing equations for phonon and phason displacements. Conservation laws for quasicrystals are utilized to derive path-independent integrals for cracks. The relation between the energy release and stress intensity factor for a crack under the mode I is given for decagonal QCs. The path-independent integral formulation is valid also for cracks in QCs with continuously varying material properties.

1. Introduction

Since the pioneering work of Eshelby [1], and independent discovery of Rice [2], innumerable number of papers have appeared in literature concerning path-independent integrals (familiary so-called J-integral) and their application to mechanics of fracture. Both papers initiated important studies, on conservation laws in finite and infinitesimal elasticity, by Knowles and Sternberg [3], and the interpretation of these in the context of the mechanics of cracks and notches in 2-dimensional bodies by Budiansky and Rice [4]. Above mentioned studies are restricted to elasto-statics, and the crack-extension considered, if any, is of quasi-static nature, i.e. inertia is considered negligible. Atluri [5] extended path-independent integrals to finite elasticity with general body forces, arbitrary crack-face tractions and nonstationary conditions. Recently, Han and Atluri [6,7] have published more general papers on conservation laws, to improve upon the work of Eshelby, Knowles, and Sternberg. The MLPG is applied together with these conservation laws there.

In many fracture problems it has been derived a unique relation between the stress intensity factors and path-independent J-integral value. Stress intensity factors characterize stress fields at the crack tip vicinity and determine fracture processes. Their evaluation from the asymptotic expansion expression of stresses is inaccurate due to numerical difficulties connected with modelling strong field gradients at the crack tip vicinity. However, the J-integral characterize fracture processes also, however, it is determined by fields sufficiently far from the crack tip. Therefore, the approach based on

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Nomenclature

Latin symbols

a	crack-length
D	kinematic coefficient of phason field
c_{ijkl}	phonon elastic tensor
H_{ij}	phason stress tensor
K_{ijkl}	phason elastic tensor
R_{ijkl}	phonon-phason coupling elastic constant tensor
n_j	outward unit normal vector
t_i	traction vector
h_i	generalized traction vector
u_i	phonon displacements
w_i	phason displacements
w_{ij}	phason strain
G	energy release rate
T	kinematic energy
U	internal energy
W	strain energy density
K_I, K_{II}	stress intensity factors

Greek symbols

ε_{ij}	strain tensor
ρ	mass density
σ_{ij}	stress tensor
τ	time
Γ	integration path
Γ_c	crack face

Other symbols

$f_{,i}$	partial derivative of the function f
$\dot{f}$	time derivative of the function f

the evaluation stress intensity factors on the J-integral is more accurate than direct approach based on asymptotic expansion of stresses. It is well known in conventional elasticity of crystals [8]. Now, it is a natural attempt to extend this idea into fracture of quasicrystals.

According to the review article [9] and monograph [10], which comprehensively give the state of the art of investigations on the mechanical analyses of QCs, the crack problems are presented in literature quite seldom. It is despite reality that quasicrystals are brittle. Therefore, to understand the effect of cracks on the mechanical behavior of a quasicrystal, the crack analysis of quasicrystals, including the determination of the stress intensity factors, the elastic field, the strain energy release rate and so on, is a prerequisite. Although a sufficiently large number of solutions associated with cracks have been obtained for various problems of theoretical and practical importance in conventional linear fracture mechanics for crystals, very few investigations are known for quasicrystals [11]. Due to the coupling of phonon and phason displacements in quasicrystals, the crack problems for these materials are more complicated than those in conventional crystals. Therefore, crack analyses in the QC are focused mainly on Griffith cracks in an infinite body, where analytical solutions are available for one and two-dimensional quasicrystals [9,12–14]. The governing equations for anti-plane crack problems are simpler than that for in-plane problems and analytical solutions are available also for the crack mode III too [15,16].

The goal of this paper is to derive path-independent integrals for crack problems in quasicrystals with nonstationary conditions. The first effort is made by Shi [17], where a special version of Noether's theorem for the sake of absolute invariance on invariant variational principles is used to obtain conservation laws of a three-dimensional solid periodically stacked in a two-dimensional quasiperiodic structure with decagonal symmetry. Conservation laws in material space depend on the material coefficients [18,19] and, indeed, may not be easy to be observed by experiment. It is well known that some path-independent integrals valid in an isotropic elastic medium does not exist in orthotropic medium. For quasicrystal, there are many kinds of symmetries of the material coefficients [20]. These material symmetries of quasicrystals should be considered to prove the existence and determine the characteristic of their conservation laws. In the present paper conservation laws for quasicrystals are utilized to derive path-independent integrals for cracks. The relation between the energy release and stress intensity factor for a crack under the mode I is given for decagonal QCs. The path-independent integral formulation is valid also for cracks in QCs with continuously varying material properties.