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Investigation into the behavior of a graded cellular rod under impact

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ABSTRACT

Over the past few decades, functionally graded materials (FGMs) have attracted many research interests. For the cellular material, the variation of the mechanical property may significantly influence the global performance of the structure. Previously the authors have presented the basic deformation patterns in a simple case, in which only the quasi-static plateaus stress gradually decreases or increases along the rod and the density is uniform. In the current study, further investigations are carried out into the effect of the gradient in the initial density, which inherently leads to variation in the quasi-static plateau stress. A simple impact scenario is considered, in which a rigid mass strikes a stationary cellular rod with variation in the initial density of the material. Similar to the previous studies, the rigid-perfectly plastic-locking (R-PP-L) material model and the simple shock theory are employed to carry out the analysis. Current study confirms that the basic deformation modes (i.e. double shock (DS) and single shock (SS) modes) still exist in the cellular rod with density gradient. The analytical results indicate that the negative gradient weakens the capacity of energy absorption while positive gradient has little influence. Then, the effect of the initial density gradient is further investigated by considering two artificial cases with only one gradient either in the quasi-static stress or in the initial density. Finally, finite element (FE) simulations are carried out with practical metal foam in order to verify the analytical modeling.

1. Introduction

Due to their excellent properties in the energy absorption, insulation and low weight, cellular materials have been widely used in the engineering applications, especially under dynamic loading conditions. Extensive experimental and theoretical research works have been carried out to study their mechanical properties under quasi-static and dynamic loadings. The mechanical properties of these materials are significantly influenced by the cell size, the material strength and the density distribution. To maximize the global mechanical property of the structure, the concept of the graded cellular material was proposed by introducing gradual variation of certain property along one direction of the cellular material.

1.1. One-dimensional shock theory

Under dynamic loading, the compaction wave in the cellular material travels from the proximal end and then extends to the distal end, leading to an enhancement in the stress and energy absorption [1,2]. Various methods have been proposed to explain such features in the dynamic loading, including the shock wave theory [1,3–7] and spring mass models [8,9]. Analytical models based on the experimental stress–strain curves were proposed by Karagiozova et al. [10]. Some debates [11–13] have taken place due to the application of the different methods. Clarification has been provided by Harrigan et al. [7] by comparing these methods of analysis for two impact scenarios, which shows the applicability of the simple shock theory. Most of the analytical models for the dynamic behavior of the cellular material were reviewed by Zhu et al. [14].

The shock theory is usually employed to model plastic shock wave propagation, which was first proposed by Reid and Peng [1] to analyze the crushing behavior of the wood specimens. The predictions have shown good agreement with the experimental results [1,6]. In this theory, the cellular material is assumed to be a rigid-perfectly plastic-locking (R-PP-L) material. Retaining the basic characteristic, Tan et al. [5,15] took into account the elastic property of the material; Harrigan et al. [6] proposed an elastic–plastic model with hardening. The model proposed by Lopatnikov et al. [3,4] also considered the elastic property but with a different definition of the locking strain. Later, researchers found that the locking strain in R-PP-L material model significantly influences the

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prediction of the shock front speed and the dynamic stress. A power law densification was incorporated by Pattofatto et al. [16] for more accurate prediction. Zheng et al. [17] took into account linearly hardening property in the plastic stage to determine the basic deformation modes in homogenous cellular rods, namely transition mode and shock mode.

The locking strain ε_d and the quasi-static plateau stress σ_p are usually obtained from a quasi-static analysis proposed by Gibson and Ashby [18]. They are expressed in terms of the relative density ρ_0/ρ_s and the yielding stress of the densified material,

$$\varepsilon_d = 1 - C\frac{\rho_0}{\rho_s} \tag{1}$$

$$\sigma_p = C_1\left(\frac{\rho_0}{\rho_s}\right)^{3/2}\sigma_{ys} \tag{2}$$

where ρ_0 and ρ_s are the initial and densified density of the cellular material, respectively; and the constants (C, C_1) are determined by experimental data. Recent studies show that the locking strain varies with the impinging velocity [16,19,20]. However, in the present paper the emphasis is on the investigation of the influence of the gradient. Therefore, the locking strain is defined as a constant for a simple analysis ($C = 1$), regardless of the impinging velocity.

In the shock theory, employing the mass and momentum conservations at the shock front, the dynamic stress σ_d in the densified zone is,

$$\sigma_d = \sigma_p + \frac{\rho_0}{\varepsilon_d}V^2 \tag{3}$$

where σ_d is the dynamic crushing stress of the cellular material and V is the velocity at the shock front. Evidently, Eq. (3) shows that changing the initial density of the cellular material influences both the quasi-static plateau stress σ_p and the dynamic enhancement term $\rho_0 V^2/\varepsilon_d$.

The experimental and theoretical analysis showed that only when the impact velocity reaches a critical value, V_{cr}, plastic compaction shock occurs. Based on wave-trapping theory, the critical velocity was defined by Honig and Stronge [21],

$$V_{cr} = \int_0^{\varepsilon_{cr}} c(\varepsilon)\mathrm{d}\varepsilon \tag{4}$$

where $c(\varepsilon) = \sqrt{(\mathrm{d}\sigma/\mathrm{d}\varepsilon)/\rho_0}$ is the speed of stress wave in a solid with a uniaxial stress–strain curve $\sigma = \sigma(\varepsilon)$, and ε_{cr} is the strain where $\mathrm{d}\sigma/\mathrm{d}\varepsilon = 0$. For a linear-elastic, perfectly plastic material, locking material (E-PP-L), $\varepsilon_{cr} = \sigma_y/E$, and Eq. (4) is reduced to,

$$V_{cr} = \frac{\sigma_y}{\sqrt{E\rho_0}} \tag{5}$$

where E and σ_y are the Young's modulus and the yielding stress, respectively. Since the Young's modulus can be regarded as infinite in the R-PP-L model, the critical velocity decreases to zero.

1.2. Graded cellular material under impact

In the recent years, the concept of the graded cellular structures has attracted much research interest. Until now, most studies were based on the finite element simulations and few experimental or analytical works were found in literature. Ali et al. [22] studied the response of a honeycomb block with different wall thickness when the impact velocity is low (up to 20 m/s). Ajdari et al. [23] employed Voronoi structures to study the uniaxial and biaxial compressive behavior. Later, they also investigated density gradient in the regular honeycomb structures [24]. Both works focused on the elastic response under low velocity impact. Shen et al. [25] studied honeycombs with gradient in the yielding strength of the material, under various impact velocities, and they summarized a map of the deformation mode. An experimental study on density gradient was carried out by Zeng et al. [26], employing the polymeric hollow sphere agglomerates with variation in wall-thickness. Their results showed that a negative gradient in the density benefits in reducing the force at the distal end. However, the basic deformation modes or the energy absorption capacity was rarely discussed. Most recently, the present authors [27] did an analytical work in the understanding of the mechanism of the response of the graded cellular under impact. The main variable is the yielding stress gradient, which changes from being positive to negative. Two basic deformation modes were identified in the graded cellular material under impact, namely, double shock and single shock modes.

1.3. Objective of this paper

The present paper focuses on the investigation into the role played by the density gradient for a cellular rod under an axial impact. The cellular material is modeled by using R-PP-L material model. The gradient is introduced by changing the initial density of the cellular material along the rod. In the beginning, a general analytical study is carried out, which gives the basic governing equations of the deformation process. Then, the influences of gradient on the characteristic parameters are analyzed, including the velocity of the impinging mass, the reaction force at the proximal and distal ends and the energy absorption capacity. Finally, finite element software ABAQUS/EXPLCIT is used to carry out numerical simulations to verify the theoretical predictions.

2. Governing equations for a graded cellular rod with density gradient

Consider a stationary cellular rod impinged by a rigid mass, G, with an initial velocity, V_0 (see Figs. 1 and 2). The material properties of the rod are uniform except for the initial density of the cellular material, which varies linearly along the rod. The initial length of the rod is l_0 and initial average density is ρ_0. The gradient in the initial density, Θ_ρ, is defined as follows,

$$\Theta_\rho = \frac{\Delta\rho_0}{\rho_0} \tag{6}$$

where $\Delta\rho_0$ is the difference in the initial density of the undeformed cellular material between the two ends of the cellular rod. The gradient is defined as positive if the initial density increases from the proximal end to the distal end.

Introducing the density gradient not only influences the inertia but also the quasi-static plateau stress. It should be noted that the R-PP-L material model, which is adopted here, is a special case of elastic-perfect plastic-locking (E-PP-L) material with the young's modulus, E approaching infinite. Thus, the elastic wave speed is infinite and the wave front reaches the distal end immediately after impact. Also, the stress of every material point reaches yielding stress after impact. Therefore, the stress wave is reflected on the far end of the rod and could build up to form a compaction wave, although it takes place in an infinitely small time interval. When the gradient is positive, the weakest part is at the proximal end and it deforms immediately after impact. However, at the