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A general approach for error modeling of machine tools

Wenjie Tian [a], Weiguo Gao [a,*], Dawei Zhang [a], Tian Huang [a,b]

[a] Key Laboratory of Mechanism Theory and Equipment Design of Ministry of Education, Tianjin University, Tianjin 300072, China
[b] School of Engineering, The University of Warwick, Coventry CV4 7AL, UK

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ABSTRACT

This paper presents a general and systematic approach for geometric error modeling of machine tools due to the geometric errors arising from manufacturing and assembly. The approach can be implemented in three steps: (1) development of a linear map between the pose error twist and source errors within machine tool kinematic chains using homogeneous transformation matrix method; (2) formulation of a linear map between the pose error twist and the error intensities of a machine tool; (3) combination of these two models for error separation. The merit of this approach lies in that it enables the source errors affecting the compensatable and uncompensatable pose accuracy of the machine tool to be explicitly separated, thereby providing designers and/or field engineers with an informative guideline for the accuracy improvement by suitable measures, i.e. component tolerancing in design, manufacturing and assembly processes, and error compensation. Two typical multi-axis machine tools are taken as examples to illustrate the generality and effectiveness of this approach.

1. Introduction

Geometric accuracy is crucially an important performance factor for machine tools, especially under circumstances where relatively high precision is one of the basic requirements [1]. There are two ways to improve the geometric accuracy of machine tools: (1) design and manufacture for precision and (2) error compensation. Both require a parametric model that relates the geometric source errors to the pose accuracy of the cutting tool relative to the workpiece. Theoretically, the effects of source errors on pose accuracy of 3-, 4- or 5-axis machine tools cannot fully be compensated by software, and only those pose errors corresponding to the permissible motion types are compensatable by means of error compensation. Therefore, a comprehensive error model is essential in order to distinguish the source errors affecting compensatable pose errors from those affecting uncompensatable ones. Only then can suitable measures be adopted for accuracy improvement [2] via component tolerance design, manufacturing and assembly, as well as by error compensation.

Over the past decades, there has been a great deal of intensive research into error modeling of machine tools. The most widely used way is based on the rigid body kinematics, in which homogeneous transformation matrix (HTM) or Denavit–Hartenberg (D–H) transformation matrix is used to represent the coordinate transformation between each rigid body frame with its reference coordinate system. Kiridena [3] classified 5-axis machine tools into TTTRR, RTTTR and RRTTT systems and used the D–H convention to develop kinematic models for each of these three machine types. However, their model considered only five parametric errors (one positioning error for each axis). Srivastava [4] described a systematic method for the development of geometrical and thermal errors based on the kinematic analysis of machine structure. However, their work only focused on one specific machine type, which is not comprehensive. Jha [5] used a generalized volumetric error model of a 5-axis machine tool based on the D–H method for geometric error compensation, which saw the quality improvement in the cam profile generation experiment. Lei [6] reported a new probe ball measurement device to measure the overall position errors of a 5-axis machine tool directly. The volumetric error modeling was carried out as a theoretical explanation based on HTM. The kinematic chain of the machine tool was closed with the probe ball rather than cutting tool workpiece during error modeling. Lin [7] proposed a new matrix summation method: the kinematic equation was converted into six components to give each component a clear physical meaning. However, the physical meaning of geometric source error was not clear. Yang [8] dedicated one chapter of his dissertation to the formulation of a generalized 5D error synthesis model. However, he defined only 27 geometric error components, which is incomplete. Many scholars [9–14] analyzed the volumetric error model of machine tool based on this method and showed its effectiveness.

Previous literature shows that most studies of error modeling mainly aimed at error compensation, and little attention has been

* Corresponding author. Tel./fax: +86 22 27405561.
E-mail addresses: tianwenjietju@163.com (W. Tian), gaowg@tju.edu.cn (W. Gao), medzhang@tju.edu.cn (D. Zhang), tianhuang@tju.edu.cn, tian.huang@warwick.ac.uk (T. Huang).

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paid to the important design question of separating the source errors into two subsets associated with the compensatable and uncompensatable pose accuracy. More recently, a novel concept of a generalized Jacobian was proposed for describing systematically the unexpected small deviations from the ideal motions of the mechanism end-effector [15]. For error modeling, the generalized Jacobian can be seen as the linear map between the error twist of the machine (6D pose error of the cutting tool relative to the workpiece) and two sets of joint error intensities associated with the permitted and restricted motions. In other words, the formulation of the generalized Jacobian allows the joint error intensities affecting the compensatable and uncompensatable pose accuracy to be separated in an explicit manner. However, finding the relationship between the joint error intensities and the geometric source errors remains a problem still to be tackled.

This paper extends our previous developments of the generalized Jacobian approach [15] to the error modeling of machine tools. Its goal is to enable explicit separation of the geometric source errors affecting the compensatable and uncompensatable pose accuracy. After Section 1 has briefly addressed current challenges in error modeling of machine tools, Section 2 introduces two kinds of geometric source errors within the kinematic chains. In Section 3, a new, systematic model is established, which formulates the linear map between the pose error twist and geometric source errors within a machine tool using the homogeneous transformation matrix method. By means of a very brief review of the concepts of the generalized Jacobian, Section 4 establishes the linear map between the pose error twist and error intensities of a machine tool. Then, combining the results of Sections 3 and 4 gives an error model that fully clarifies which source errors affect the compensatable and uncompensatable pose accuracy. Finally, Section 5 takes two typical machine tools as examples to illustrate the generality and effectiveness of this approach before conclusions are drawn in Section 6.

2. Geometric errors of machine tool

The geometric error of the machine tool refers to the error of individual axes and those between axes. The error components are commonly expressed as positioning errors, straightness errors, angular errors and squareness errors. From the perspective of rigid body kinematics, every translational axis and rotational axis has six geometric error components that are position dependent, which contain linear and angular errors.

In a translational axis, the six error components are one positioning error, two straightness errors, and three angular errors called pitch, yaw and roll respectively (see Fig. 1). In a rotational axis, the three linear error components are one axial error and two radial errors, and the three angular error components are one angular position error and two tilt errors (see Fig. 1). All the possible position-dependent errors of machine tools are shown in Table 1. δ_x, δ_y and δ_z represent the linear errors, where the subscript represents the error direction. ε_x, ε_y and ε_z represent the angular errors, where the subscript represents the rotation axis of angular error. x, y and z are the linear motion coordinate values of X-axis, Y-axis and Z-axis, respectively, while α, β and γ are the rotation angles of A-axis, B-axis and C-axis, respectively.

There are also position-independent errors in the machine tool, which are the location errors of an axis defined as an error from the nominal position and orientation of this axis in the machine coordinate system. Theoretically, a translational/rotational axis has six location errors. However, most of the location errors can be neglected in real error modeling due to the selection of reference

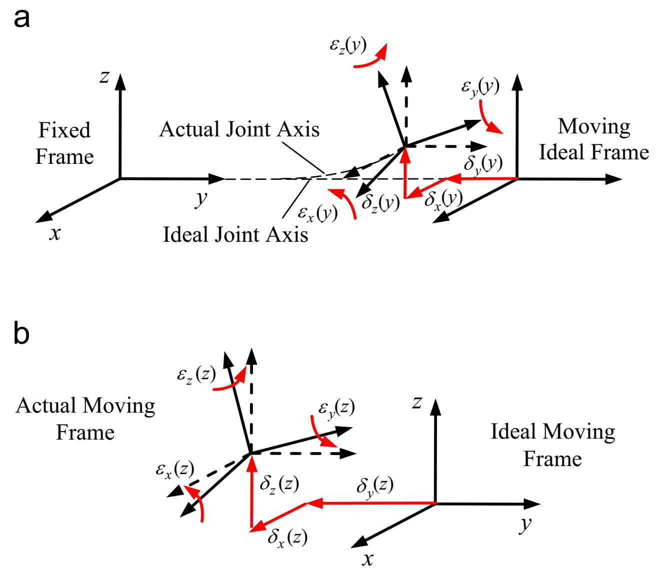

Fig. 1. Linear and angular errors of a joint. (a) Translational Axis (Y Axis) and (b) Rotational Axis (Z Axis).

Table 1
Position-dependent errors of machine tools.

Axis	Error components
X-axis	$\delta_x(x),\delta_y(x),\delta_z(x),\varepsilon_x(x),\varepsilon_y(x),\varepsilon_z(x)$
Y-axis	$\delta_x(y),\delta_y(y),\delta_z(y),\varepsilon_x(y),\varepsilon_y(y),\varepsilon_z(y)$
Z-axis	$\delta_x(z),\delta_y(z),\delta_z(z),\varepsilon_x(z),\varepsilon_y(z),\varepsilon_z(z)$
A-axis	$\delta_x(\alpha),\delta_y(\alpha),\delta_z(\alpha),\varepsilon_x(\alpha),\varepsilon_y(\alpha),\varepsilon_z(\alpha)$
B-axis	$\delta_x(\beta),\delta_y(\beta),\delta_z(\beta),\varepsilon_x(\beta),\varepsilon_y(\beta),\varepsilon_z(\beta)$
C-axis	$\delta_x(\gamma),\delta_y(\gamma),\delta_z(\gamma),\varepsilon_x(\gamma),\varepsilon_y(\gamma),\varepsilon_z(\gamma)$

position and coordinate system. Only orientation errors will be left, which mean squareness errors.

3. Error modeling

Error modeling aims to establish a map from geometric source errors to the pose errors of machine tool, which is the common premise of precision design and error compensation. All kinds of machine tools have two structural chains which contain several components connected in series by prismatic and rotational joints. One is from machine bed to workpiece, and the other is from machine bed to cutting tool, which are represented by workpiece kinematic chain and cutting tool kinematic chain, respectively. Thus, establishing the error model of kinematic chain is the foundation of the whole machine tool error model.

3.1. Error modeling for kinematic chain

Fig. 2 shows the nominal/actual configurations of the f-DOF kinematic chain without/with accounting for geometric source errors. Modeling the chain error uses two global reference frames R and R': R is located at point O on the machine bed while R' is located at point O' on the tool or workpiece and remains parallel to R. In order to describe the source errors within the chain and relate them to pose accuracy of tool/workpiece, body-fixed frame R_{f+1} is placed at point O', and body-fixed frames R_i are placed on the ith joint ($i=1,2,\ldots,f$). As previously mentioned, the source errors can be divided into two categories: