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Short Communication

Least squares approximated stability boundaries of milling process

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ABSTRACT

First and second order least squares methods are used in generating simple approximation polynomials for the state term of the model for regenerative chatter in the milling process. The least squares approximation of delayed state term and periodic term of the model does not go beyond first order. The resulting discrete maps are demonstrated to have same convergence rate as the discrete maps in other works that are based on the interpolation theory. The presented discrete maps are illustrated to be beneficial in terms of computational time (CT) savings that derive from reduction in the number of calculation needed for generation system monodromy matrix. This benefit is so much that computational time of second order least squares-based discrete map is noticeably shorter than that of first order interpolation-based discrete map. It is expected from analysis then verified numerically that savings in CT due to use of least squares theory relative to use of interpolation theory of same order rises with rise in order of approximation. The experimentally determined model parameters used for numerical calculations are extracted from literature.

1. Introduction

Chatter is the self-excited vibration of tool-workpiece interaction in machine tools. Some of the known popular causes of chatter are regenerative effects [1,2,3,4], mode coupling effects [5,6], frictional effects [7] and thermo-mechanical effects [8]. Regenerative effects which were first suggested as a potential cause of chatter by Arnold [9] is now considered the most common cause of chatter. Regenerative effects are the perturbation-induced waviness on a machined surface. Random nature of perturbations causes two consecutive tool passes to be out of phase resulting in cutting force variation that excites the tool. The resulting chatter grows if cutting parameter combination is unstable but remains bounded if the cutting parameter combination is stable. For this reason, most analysis on regenerative chatter is geared towards determining the stability lobe that separates stable cutting domain from the unstable domain. A method that achieves this demarcation by utilizing the mean of the Fourier series of the dynamic milling coefficients called the Zeroth Order Approximation (ZOA) method was proposed by Altintas and Budak [1] and upgraded for use in three dimensional chatter stability analysis by Altintas [10]. Even though the ZOA method is fast it lacks the capacity to accurately predict stability at low radial immersions [11,12]. The other methods that follow ZOA are strongly based on the Floquet theory. They seek a linear operator called Floquet transition matrix that transforms the whole delayed state to the whole present state. Stability lobe is then computed from eigen-value analysis of the resulting Floquet transition matrix. These methods have the capacity lacking in the ZOA method in that they can predict stability in both high and low-radial immersions. Temporal Finite element analysis (TFEA) is one of these methods that originally seemed to have a shortcoming opposite to that of the ZOA method in that it (TFEA) failed to predict accurate stability at high-radial immersion and cuts involving simultaneous tooth engagement [13]. Increasing the size of Floquet transition matrix of TFEA by simply increasing the number of elements of discrete delay was seen to solve the problem [14,15]. The semi-discretization method was introduced by Insperger and Stepan [16] for analysis of delayed systems. Some of the other works utilizing the semi-discretization method in milling chatter stability analysis are [17,18,19,20]. So far the semi-discretization method has not been proven to have any major problem in milling chatter stability analysis except that the recently developed method called the full-discretization method [21,22,23] proved to save more computational time. Interpolation polynomials are introduced in the integration scheme of the full-discretization and solved to produce a discrete map used for stability analysis in the works [21,22,23]. In the present study, least squares approximation theory is used instead of interpolation theory. It is seen that the presented use of least squares approximation is very promising in terms of further computational time savings.

2. General least squares method

The least squares method approximates a set of known responses $\boldsymbol{x}_i$ with a function $\boldsymbol{x}(\boldsymbol{z})$ by minimizing the sum of squares of Euclidian error norms $\|\boldsymbol{x}(\boldsymbol{z}_i)-\boldsymbol{x}_i\|$. The n response vectors $\boldsymbol{x}_i$ are

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located at positions $\boldsymbol{z}_i$ in a real d-dimensional space of independent variables. Symbolically $\boldsymbol{z}_i \epsilon R^d$ where $i\epsilon[1, 2, \ldots..n]$. The approximation function $\boldsymbol{x}(\boldsymbol{z})$ is usually a polynomial of form

$$\boldsymbol{x}(\boldsymbol{z}) = [\boldsymbol{a}(\boldsymbol{z})]^T \boldsymbol{b} \tag{1}$$

where the polynomial of basis vector $\boldsymbol{a}(\boldsymbol{z}) = \{a_1(\boldsymbol{z})\ a_2(\boldsymbol{z}) \cdots\cdots a_l(\boldsymbol{z})\}^T$ contains the independent variables and the vector of coefficients $\boldsymbol{b} = \{b_1\ b_2 \ldots\ldots b_l\}^T$ contains the coefficients of approximation function $\boldsymbol{x}(\boldsymbol{z})$. For illustration the linear scalar bivariate approximation function has dimension $d=2$ and order $p=1$ such that

$$\boldsymbol{x}(\boldsymbol{z}) = x(z_1, z_2) = b_1 + b_2 z_1 + b_3 z_2$$

$$\boldsymbol{x}(\boldsymbol{z}) = x(z_1, z_2) = \{ 1 \quad z_1 \quad z_2 \} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

from which it is seen that $\boldsymbol{a}(\boldsymbol{z}) = \{ 1 \quad z_1 \quad z_2 \}^T$ and $\boldsymbol{b} = \{ b_1 \quad b_2 \quad b_3 \}^T$ and length $l=3$. The length of $\boldsymbol{a}(\boldsymbol{x})$ or $\boldsymbol{b}$ is generally given as $l = \frac{(d+p)!}{d!p!}$. The least squares method boils down to minimizing the error functional

$$E(\boldsymbol{b}) = \sum_{i=1}^{n} \|\boldsymbol{x}(\boldsymbol{z}_i) - \boldsymbol{x}_i\|^2 \tag{2}$$

Approximation function $\boldsymbol{x}(\boldsymbol{z}_i) = [\boldsymbol{a}(\boldsymbol{z}_i)]^T \boldsymbol{b}$ at the known locations are inserted in Eq. (2) to give

$$E(\boldsymbol{b}) = \sum_{i=1}^{n} \|[\boldsymbol{a}(\boldsymbol{z}_i)]^T \boldsymbol{b} - \boldsymbol{x}_i\|^2 \tag{3}$$

The coefficient vector $\boldsymbol{b}$ at which $E(\boldsymbol{b})$ is minimized is determined by differentiating with respect to $\boldsymbol{b}$ and equated to zero to give

$$\frac{\partial}{\partial \boldsymbol{b}} E(\boldsymbol{b}) = \sum_{i=1}^{n} 2\boldsymbol{a}(\boldsymbol{z}_i) \left\{ [\boldsymbol{a}(\boldsymbol{z}_i)]^T \boldsymbol{b} - \boldsymbol{x}_i \right\} = 0 \tag{4}$$

Eq. (4) is expanded to give

$$\sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{z}_i)[\boldsymbol{a}(\boldsymbol{z}_i)]^T \boldsymbol{b} - \sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{z}_i)\boldsymbol{x}_i = 0 \tag{5}$$

The minimum-error coefficient vector $\boldsymbol{b}$ is obtained through matrix inversion process

$$\boldsymbol{b} = \left\{ \sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{z}_i)[\boldsymbol{a}(\boldsymbol{z}_i)]^T \right\}^{-1} \sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{z}_i)\boldsymbol{x}_i \tag{6}$$

Eq. (6) is inserted in Eq. (1) to give the approximation polynomial as

$$\boldsymbol{x}(\boldsymbol{z}) = [\boldsymbol{a}(\boldsymbol{z})]^T \left\{ \sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{z}_i)[\boldsymbol{a}(\boldsymbol{z}_i)]^T \right\}^{-1} \sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{z}_i)\boldsymbol{x}_i \tag{7}$$

Some more fundamental about least squares method can be gained from the works [24,25,26]. In making use of Eq. (7), the summation sign is considered a multiplying factor to each of the elements of the matrix $\boldsymbol{a}(\boldsymbol{z}_i)[\boldsymbol{a}(\boldsymbol{z}_i)]^T$ and vector $\boldsymbol{a}(\boldsymbol{z}_i)\boldsymbol{x}_i$. Eq. (7) can be verified by looking back at the earlier mentioned illustrative linear scalar bivariate approximation function. It will be seen that taking the partial derivatives of the error functional with respect to the coefficients b_1, b_2 and b_3 gives a linear system of equation that could be re-arranged to give same result as that of the direct use eq. (7).

3. Least squares application in milling discrete mapping

The full-discretization method [21] requires that state space equation governing regenerative milling process with single discrete delay is put in the form

$$\dot{\boldsymbol{\xi}}(t) = \mathbf{A}\boldsymbol{\xi}(t) + \mathbf{B}(t)\boldsymbol{\xi}(t) - \mathbf{B}(t)\boldsymbol{\xi}(t-\tau) \tag{8}$$

where $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}$ is a constant matrix that that contains the time-invariant parameters of the system and $\mathbf{B}(t) = \begin{bmatrix} 0 & 0 \\ -\frac{wh(t)}{m} & 0 \end{bmatrix}$ is a coefficient matrix that captures the periodicity of cutting force of the unperturbed milling process. Periodicity of $\mathbf{B}(t)$ stems from periodicity of the specific force variation $h(t)$ given as

$$h(t) = \gamma(v\tau)^{\gamma-1} C_{\tan} \sum_{j=1}^{N} g_j(t) sin^{\gamma}\theta_j(t)\left[\chi sin\theta_j(t) + cos\theta_j(t)\right] \tag{9}$$

The non-linear tangential and normal cutting force models used in deriving eq. (8) are of the forms [28]

$$\begin{aligned} F_{\tan,j}(t) &= C_{\tan} w\left[f_a \sin\theta_j(t)\right]^{\gamma} \\ F_{\text{norm},j}(t) &= C_{\text{norm}} w\left[f_a \sin\theta_j(t)\right]^{\gamma} = \chi F_{tan,j}(t) \end{aligned} \tag{10}$$

the full derivation of Eq. (8) can be seen in [24,4]. The parameters of Eq. (10) are; w is the depth of cut, $C_{\tan}$ and C_{norm} are the tangential and normal cutting coefficient, χ is the ratio $C_{\text{norm}}/C_{\tan}$, f_a, is the actual feed and γ is an exponent that is not greater than one.

The discrete delay τ of the system is divided into k equal discrete time intervals $[t_i, t_{i+1}]$ where $i = 0, 1, 2, \ldots\ldots..(k-1)$ and $t_i = i\frac{\tau}{k} = i\Delta t = i(t_{i+1} - t_i)$. Eq. (8) is represented in the discrete interval $[t_i, t_{i+1}]$ as

$$\dot{\boldsymbol{y}}(t) = \mathbf{A}\boldsymbol{y}(t) + \mathbf{B}(t)\boldsymbol{y}(t) - \mathbf{B}(t)\boldsymbol{y}(t-\tau) \tag{11}$$

Eq. (11) is integrated between the limits t_i and t_{i+1} to become

$$\mathbf{y}_{i+1} = e^{\mathbf{A}\Delta t}\mathbf{y}_i + \int_{t_i}^{t_{i+1}} e^{\mathbf{A}(t_{i+1}-s)}[\mathbf{B}(s)\boldsymbol{y}(s) - \mathbf{B}(s)\boldsymbol{y}(s-\tau)]\mathrm{d}s \tag{12}$$

Least squares method is then used to approximate the terms $\boldsymbol{y}(s), \boldsymbol{y}(s-\tau)$, and $\mathbf{B}(s)$.

3.1. First-order least squares approximation

The first-order or linear least squares approximation of the state term $\boldsymbol{y}(s)$ is seen from Eq. (7) to become

$$\boldsymbol{y}(s) = \{1\ s\}\left[\sum_{l=i}^{i+1} \begin{Bmatrix} 1 \\ s_l \end{Bmatrix} \{1\ s_l\}\right]^{-1} \sum_{l=i}^{i+1} \begin{Bmatrix} 1 \\ s_l \end{Bmatrix} \mathbf{y}_l \tag{13}$$

It should be noted that $l=i$ and $l=i+1$ in the summation signs of Eq. (13) respectively correspond to terms at t_i and t_{i+1}. This is re-written as

$$\boldsymbol{y}(s) = \{1\ s\}\begin{bmatrix} \sum_{l=i}^{i+1} 1 & \sum_{l=i}^{i+1} s_l \\ \sum_{l=i}^{i+1} s_l & \sum_{l=i}^{i+1} s_l^2 \end{bmatrix}^{-1} \begin{Bmatrix} \sum_{l=i}^{i+1} \mathbf{y}_l \\ \sum_{l=i}^{i+1} s_l \mathbf{y}_l \end{Bmatrix} \tag{14}$$

Eq. (14) is further re-written to become

$$\boldsymbol{y}(s) = \frac{\{1\ s\}}{\sum_{l=i}^{i+1} 1 \sum_{l=i}^{i+1} s_l^2 - \left(\sum_{l=i}^{i+1} s_l\right)^2} \begin{bmatrix} \sum_{l=i}^{i+1} s_l^2 & -\sum_{l=i}^{i+1} s_l \\ -\sum_{l=i}^{i+1} s_l & \sum_{l=i}^{i+1} 1 \end{bmatrix} \begin{Bmatrix} \sum_{l=i}^{i+1} \mathbf{y}_l \\ \sum_{l=i}^{i+1} s_l \mathbf{y}_l \end{Bmatrix} \tag{15}$$