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Interlaminar stress analysis of piezo-bonded composite laminates using the extended Kantorovich method

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ABSTRACT

An iterative method has been applied to analyze the free edge interlaminar stresses of piezo-bonded composite laminates. Electric fields are applied to two symmetrically bonded piezo actuators that can generate induced strain, resulting in pure extension of the whole structure. The stresses, which satisfy the traction-free and free edge boundary conditions, are obtained by taking the principle of complementary virtual work, and conducting the extended Kantorovich method. In order to obtain accurate interlaminar stress distributions, static and kinematic continuity conditions are applied at the interfaces between plies through iterations. The stress components were obtained under the plane strain assumption as well as antiplane shear assumption. The results were compared with those obtained by the finite element method, to demonstrate the validity of the proposed method. Through the iteration processes, the interlaminar stresses converged well, and predicted maximum peak stress at the interface of piezo and composite layers. The present method provides accurate stresses distribution near the free edges, of piezo-bonded composite laminates.

1. Introduction

Composite laminates with piezoelectric layers can be considered as smart structures. Such smart structures are capable of sensing and actuating in various fields. Commonly, they are widely used in the energy harvesting industries, and the control of dynamic motions with their precise nature [1–3]. Advantages of composite laminates, such as high strength, high stiffness and light weight, are well inherited in these smart structures. Thus, the study and application of smart structures are becoming more and more popular. Until now, numerous research efforts related to modeling of piezo-bonded composite laminates have been fulfilled in both static and dynamic aspects [4,5]. A huge volume of theories have been advanced, and some of them have been verified by experiment data.

Stress concentrations or singularity phenomena near the free edges have been recovered by previous work as the main cause of delamination of the whole structure. Due to the mismatch of material properties between plies, edge effects are critical to the strength and fatigue of laminated structures. The debonding of piezo layers and crack initiation near the free edge are the most common failures in laminated structures. However, to obtain the exact singular elasticity solutions of free edge interlaminar stress problems still shows great difficulties; only approximate methods have been developed, based on either displacement fields or stress fields [6–9].

There is a large volume of the literature sources introducing the study of interlaminar stresses of piezo-bonded composite laminates using displacement fields based approaches. Among the displacement fields based approaches, classical laminate theory (CLT) [10,11], three equivalent single-layer shear deformation theories (ESLSDT) [12–14], layerwise shear deformation theory (LWSDT) [15–18], refined or enhanced shear deformation theories [19,20], and finite element method (FEM) [21–23] are the most popularly used approaches. These theories are further extended to dynamic investigations [24,25], and optimal design of composite layered structures [26]. CLT neglects transverse shear deformation, which is necessary for moderately thick laminates. ESLSDT assumes the approximate global displacement fields with a specified order, but continuous displacement fields cannot recover transverse shear stress continuity through the thickness by using constitutive relations. Displacement field based LWSDT adopts layerwise displacement fields, and satisfies not only displacement continuity conditions, but also stress continuity conditions. Nonetheless, the requirement of an extra number of degrees of freedom in the assumed displacement fields increases the complexity of calculation.

Other than displacement based theory, stress function based theory is also a widely used theory nowadays. The assumed admissible stress functions must satisfy force and moment equilibrium equations, and

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traction free boundary conditions [27]. This approach also consists of equivalent single-layer theory [28–30], and layerwise theory; both of them are based on initially assumed admissible stress functions. Yin [31,32] proposed a variational method using piecewise polynomial approximations for out-of-plane stress functions, based on stress-based layerwise theory (SBLWT). The stress functions involved in his approach not only satisfy the pointwise equilibrium equation, but are also continuous over each layer, and at the layer interfaces. Flanagan [33] proposed a solution method based on a series expansion of eigenmode shapes of a clamped–clamped beam, for determining the free-edge stresses in composite laminates. His approach can be concluded into stress based equivalent single-layer theory (SBESLT) that is computationally more efficient than Yin's work. However, the out-of-plane stress distributions were not predicted accurately, and undesired oscillations appeared, due to the initial assumption of harmonic functions. The accuracy of his approach also largely depended on the number of initially assumed stress functions.

To accurately predict the interlaminar stress distributions of piezo-bonded composite laminates, stresses should satisfy the boundary conditions at the free edge, and at the top and bottom surfaces. Stress continuity conditions should also be satisfied at the ply interfaces. Thus, in the presence of the above properties of stresses, we prefer using the stress function based extended Kantorovich method [34,35], which is an iterative method. The solution procedure is conducted under the assumptions of both plane strain state and antiplane shear state. This method mainly consists of two steps. In the first step, global out-of-plane stress fields are assumed; by taking the principle of complementary virtual work, and applying boundary conditions, the governing equations are obtained with respect to the coupled in-plane stress functions. After solving the governing equations, interlaminar stresses can be obtained from the combination of the assumed out-of-plane stress functions and calculated in-plane stress functions. However, interlaminar stresses obtained from the first step cannot guarantee accurate interlaminar stress distributions. Thus, one more step can be conducted. During the second step, layer independent out-of-plane stress functions are adopted, and stress continuity conditions are enforced at ply interfaces. By this procedure, interlaminar stress distributions can be improved, and initially presented undesirable oscillations which can also be significantly eliminated. The difference from layerwise theory is that our proposed method is an iterative method. Small terms of the initially assumed stress functions could lead to accurate results. Although the initially assumed stress functions largely influence the final accuracy, in our approach, more processes can be conducted, until convergent results are obtained.

2. Mathematical formulations

The geometric configuration of piezo-bonded composite laminate is shown in Fig. 1. Two piezoelectric actuators are assumed to be perfectly bonded on the top and bottom surfaces of laminate. The same electric fields are applied to both actuators, resulting in pure extension, due to the electro–mechanical coupling effect.

The linear elastic constitutive equations are assumed for each ply, and expressed in the following form:

$$\{\varepsilon_i\} = \left[\bar{S}\right]^{(k)}\{\sigma_i\} + \left[d\right]^{(k)}\{E\}^{(k)}, \quad (i = 1, 2, ..., 6) \tag{1}$$

where $\{\varepsilon_i\}$ and $\{\sigma_i\}$ represent strain and stress vectors, respectively. $[\bar{S}]$ is the generalized compliance matrix for orthotropic materials. $[d]$ is the piezoelectric constant matrix, and $[E]$ is the applied electric field. The superscript (k) refers to the k-th ply in the laminate. Details of these matrices are presented in Eqs. (A.1)–(A.5).

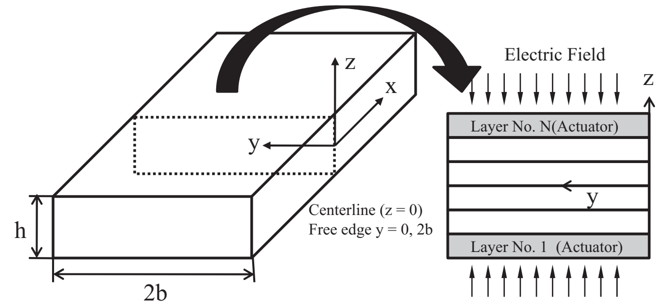

Fig. 1. Geometric configuration of piezo-bonded composite laminate with applied electric field.

If the piezo-bonded laminated composite is long enough, the stress fields are independent of the x-axis. Therefore, generalized plane strain states are assumed in the x direction. Under these assumptions, the following relation can be obtained from the first row of Eq. (1)

$$\sigma_1 = \frac{\varepsilon_1 - S_{1j}^{(k)}\sigma_j - d_{13}^{(k)}E_3^{(k)}}{S_{11}^{(k)}} \tag{2}$$

All strain components can be expressed using $\{\varepsilon_1\}$ and the applied piezoelectric loading, by substituting Eq. (2) into Eq. (1)

$$\varepsilon_i = \hat{S}_{ij}^{(k)}\sigma_j + \frac{S_{i1}^{(k)}}{S_{11}^{(k)}}\varepsilon_1 + \hat{d}_{i3}^{(k)}E_3^{(k)}, \quad (i = 2, 3, ..., 6, \ j = 2, 3, ..., 6) \tag{3}$$

where

$$\hat{S}_{ij}^{(k)} = S_{ij}^{(k)} - \frac{S_{i1}^{(k)}S_{1j}^{(k)}}{S_{11}^{(k)}}, \quad \hat{d}_{i3}^{(k)} = d_{i3}^{(k)} - \frac{S_{i1}^{(k)}}{S_{11}^{(k)}}d_{13}^{(k)} \tag{4}$$

Lekhnitskii stress functions [36] are introduced, to analyze the three dimensional stress state of piezo-bonded composite laminates under electric excitation. Lekhnitskii stress functions automatically satisfy the pointwise equilibrium equations

$$\sigma_2 = \frac{\partial^2 F}{\partial \eta^2}, \quad \sigma_3 = \frac{\partial^2 F}{\partial \xi^2}, \quad \sigma_4 = -\frac{\partial^2 F}{\partial \xi \partial \eta}$$
$$\sigma_5 = -\frac{\partial \psi}{\partial \xi}, \quad \sigma_6 = \frac{\partial \psi}{\partial \eta} \tag{5}$$

where $\eta = z/h$ and $\xi = y/h$ are nondimensionalized coordinates. These stress functions can be divided into in-plane and out-of-plane stress functions.

$$F = \sum_{i=1}^{n} f_i(\xi)g_i(\eta), \quad \psi = \sum_{i=1}^{n} p_i(\xi)g_{i,\eta}(\eta) \tag{6}$$

where $f_i(\xi)$ and $p_i(\xi)$ are in-plane stress functions, and $g_i(\eta)$ is the out-of-plane stress function. The subscript η in Eq. (6) denotes the order of differentiation with respect to η. If the in-plane and out-of-plane stress functions are defined under the prescribed loading conditions, the interlaminar stresses of the composite laminates could be analyzed. To obtain these stress functions under the piezoelectric excitation, the extended Kantorovich method is proposed in this paper. The extended Kantorovich method is an iterative method to approximate these stress functions, and two processes with detailed calculations are presented in this paper.

2.1. 1st process: calculation of interlaminar stresses by assuming the out-of-plane stress function

In the 1st process, the out-of-plane stress function $g_i(\eta)$ is first assumed to satisfy the traction free boundary conditions on the top and bottom surfaces of the piezo-bonded composite laminates. The mode shape functions of a clamped–clamped beam are assumed as the initial out-of-plane stress functions, which can