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Stresses around hypotrochoidal hole in infinite isotropic plate

Dharmendra S. Sharma

Faculty of Technology and Engineering, The M S University of Baroda, Vadodara 390001, India

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ABSTRACT

A solution to obtain stresses around hypotrochoidal cutouts in infinite isotropic plate subjected to in-plane loading is presented. The Muskhelishvili's complex variable approach is adopted to find stresses and stress intensity factors for different shaped hypotrochoidal holes. The effect of hole geometry and loading conditions on stress pattern is presented. Some of the results are compared with the existing literature and found to be in good agreement.

1. Introduction

In general, the holes and cutouts adversely alter stress field in the domain and results in strength degradation. But, the cutouts are inevitable for certain service/operational requirements. Depending upon the requirements, a cutout may be circular, elliptical, triangular, rectangular or any other regular or irregular shape.

Kirsch [1] gave solution for the isotropic plate with a circular hole using real variables. Since then the research in the field of stress distribution around holes and cutouts is ongoing. Exhaustive literature is available for regular shaped cutouts (Circular, elliptical, triangular, rectangular, and oval) in isotropic/anisotropic media.

The stress field around polygonal ([2–8]) and irregular shaped holes ([9,10], etc.) in infinite plate are found and effect of various parameters like hole geometry, material properties and loading conditions is presented. The majority of the hole shapes considered by the above authors have finite radius at the corners (non-singular), hence stress concentration factors are sufficient to describe the elevation of the stresses around the discontinuity. The hole corner may have zero radius (singular point) in some of the cases. In such cases, stress intensity factors are to be evaluated. The stress intensity factor at the cusp of hypocycloidal hole in infinite isotropic ([11,12]) and anisotropic ([13]) plate is presented. The load angle, material parameter and number of cusp points affect the stress intensity factors.

In this paper, a generalized solution is presented using Muskhelishvili's [14] complex variable approach for an infinite isotropic plate with hypotrochoidal holes. The hypotrochoidal curve family presented here includes crack, circular hole, elliptical hole, polygonal hole and hypocycloidal hole derived from a single mapping function. The stress distribution around holes with rounded corners and stress intensity factors for hole with singularity are presented. Some of the results are compared with the existing literature and found to be in close agreement.

2. Mathematical formulation

The basic equations of plane elasticity in complex variable form are given by Kolosov–Muskhellishvilli [14] as follows.

$$\sigma_x+\sigma_y=2\left[\varphi'(z)+\overline{\varphi'(z)}\right]=4\Re e[\varphi'(z)] \quad (1)$$

$$\sigma_y-\sigma_x+2i\tau_{xy}=2\left[\bar{z}\varphi''(z)+\psi'(z)\right] \quad (2)$$

where $\varphi(z),\psi(z)$ are the complex potentials of the complex variable.

2.1. Mapping function

A hypotrochoid is a curve traced by a point P attached to a circle (rolling circle) rolling without slipping inside of a fixed circle (Directing circle). The radii of the rolling and directing circle are 'r' and 'R' respectively and the distance of the tracing point from the center of the rolling circle is 'h'.

E-mail address: dss_iit@yahoo.com

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The parametric equation for a hypotrochoid can be written as

$$x(\theta) = (R-r)\cos\theta + h\cos\left(\frac{R-r}{r}\theta\right) \quad (3)$$

$$y(\theta) = (R-r)\sin\theta - h\sin\left(\frac{R-r}{r}\theta\right) \quad (4)$$

The area outside the hypotrochoidal hole (in z-plane) can be mapped to the region outside the unit circle (origin at $\zeta=0$) in ξ-plane using following mapping function developed using parametric equations mentioned above (Eqs. (3) and (4)).

$$z = \omega(\zeta) = (R-r)\zeta + h\zeta^{\frac{r-R}{r}} = R\left(\left(1-\frac{1}{N}\right)\zeta + \frac{h}{R}\frac{1}{\zeta^{N-1}}\right);$$

$$0 \le h \le r;\ N = R/r;\ N \ge 2. \quad (5)$$

Table 1
Different hole shapes obtained from the mapping function.

S. no.	N	h	Hole shape	Hole size factor (L_1/R)	Remark
1	$N \ge 3$	$h = r$	Hypocycloid with 'N' number of cusp	1	The hypocycloid is inscribed in a circle having radius equal to R
2	$N \ge 3$	$h = 0$	circle	$\frac{R-r}{R}$	radius $= R-r$
3	$N \ge 3$	$0 < h < r$	hypotrochoids with N sides	$\frac{R-r+h}{R}$	The hypotrochoids will be inscribed in a circle having radius equal to $R-r+h$
4	$N = 2$	$h = r$	Straight line (Crack)	1	Crack length $= 2R$
5	$N = 2$	$h = 0$	Circle	$\frac{r}{R}$	Radius $= r$
6	$N = 2$	$0 < h < r$	Ellipse	$\frac{r+h}{R}$	Semi-major axis $a = 0.5R + h$ semi minor axis $b = 0.5R - h$

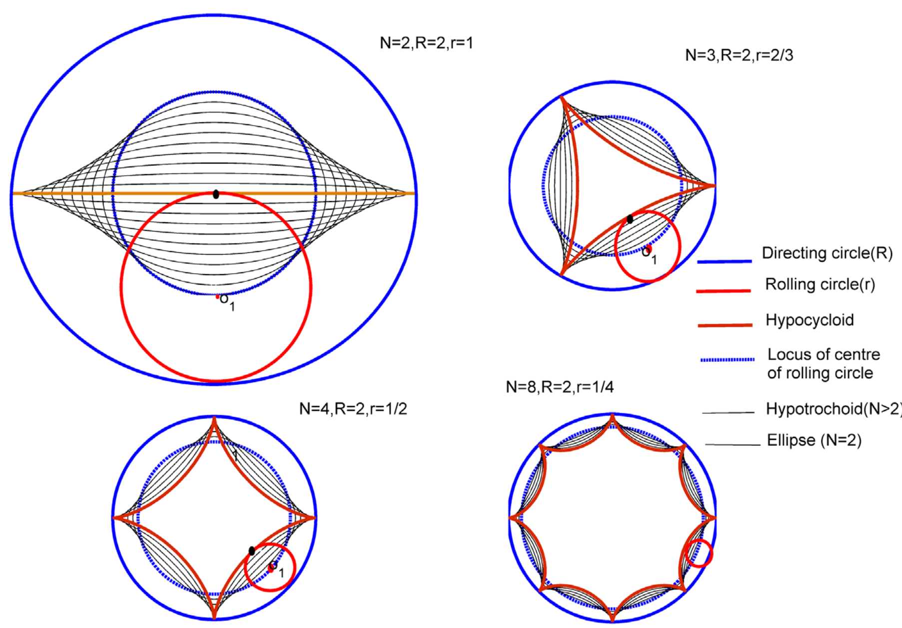

Fig. 1. Hypotrochoidal curves for N=2,3,4 and 8.

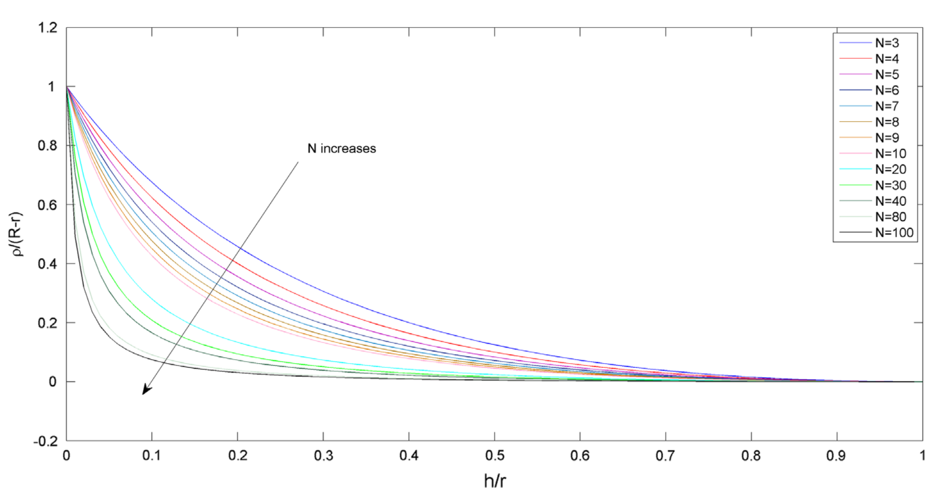

Fig. 2. Relationship between normalized corner radius and h/r.