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Time/wave domain analysis for axially moving pre-stressed nanobeam by wavelet-based spectral element method

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ABSTRACT

In this article, a time/wave domain analysis is presented for an axially moving pre-stressed nanobeam by wavelet-based spectral element (WSE) method. WSE scheme is constructed as spectral element method (SEM), except that Daubechies wavelet basis functions are used for transforming governing partial differential equation. These basis functions help to rule out some deficiencies of SEM due to periodicity assumption, especially for time domain analysis. Numerical examples are used for validating the accuracy and efficiency of model. The higher accuracy of WSE approach is then evaluated by comparing its results with those of classical finite element and SEM. The effects of moving nanobeam properties, such as velocity, pretention and nonlocal (small-scale) parameter, on vibration and wave characteristics and dispersion curves are investigated. In addition, the instability of moving nanobeam is studied both analytically and numerically considering divergence and flutter.

1. Introduction

Axially moving structures and continua can be found in numerous engineering devices in mechanical, civil, electrical and aerospace applications, *e.g.*, thread-lines in fabric industry, rolled steel beams, chain and belt drives, high-speed sheets, magnetic tapes, band saw blades, aerial cable tramways and so on. Due to this pervasiveness, the analysis of moving structures and continua has been a motivation for a large amount of publications, mostly on axially moving beams. The lateral vibration of such axially moving systems is commonly modeled as a pre-stressed beam. It is necessary to forecast the dynamic characteristics of such systems precisely, in order to reach safe, reliable and successful designs, while hazards and accidents are prevented.

There are many articles which analyze axially moving macro-scale (classical) beams. The solutions of equation of motion for moving classical beam models were obtained by several solution techniques, containing the Galerkin's [1–3], assumed modes [4], finite element [5], Green's function [6], transfer function [7], perturbation [8], the Laplace transform [9], artificial parameters [10] and the FFT-based spectral element methods (SEM) [11]. Despite the large number of studies for axially moving classical beams, few analyses have been conducted on similar problems at micro- and nano-scales. Assumed modes method and Galerkin approach [12], higher-order strain gradient solutions [13], and modified couple stress theory [14] are used to investigate moving micro- and nano-scale beam models.

As a versatile numerical method, the FEM has an important role in structural analysis. This method may provide accurate dynamic response of a structure if the wavelength is large compared to the mesh size. However, the FEM results become increasingly inaccurate as the frequency bandwidth increases. As a drawback of FEM, it is well known that a large number of FE's should be generated to obtain trustworthy and accurate solutions, especially at higher frequencies. Evidently, this provision may increase the computational time and cost. These problems are generally resolved by SEM. SEM [15] transforms the governing partial differential equation (PDE) of motion to a set of ordinary differential equations (ODE's) by FFT. These space-dependent ODE's are solved exactly, which are then used as dynamic shape functions for SEM. It is well known that the wavelet-based spectral element (WSE) model is an exact solution method for dynamic analysis of structures [16–18]. WSE formulation is very similar to SEM formulation, except that Daubechies wavelet basis functions are used for transformation of governing PDE. The ensuing coupled ODE's can be decoupled by performing a wavelet-dependent

Abbreviations: AD, artificial damping; BVP, boundary-value problem; CNT, carbon nanotube; FEM, finite element model; NP/BVP, non-periodic boundary-value problem; ODE's, ordinary differential equations; P/BVP, periodic boundary-value problem; PDE, partial differential equation; P/WSE (P/WSFE), periodic wavelet-based spectral (finite) element; SEM (SFEM), spectral (finite) element method; WAD, weak artificial damping; WSE (WSFE), wavelet-based spectral (finite) element

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Nomenclature

c	constant transport speed [m/s]
c_D	divergence speed [m/s]
c_F	flutter speed [m/s]
e_0a	nonlocal (small-scale) parameter [nm]
EI	flexural rigidity [Nm^2]
$f(x,t)$	excitation force [N]
f_c	cut-off frequency [Hz]
f_{nyq}	Nyquist frequency [Hz]
$\{\widehat{F}^g\}$	global WSE nodal forces [N, Nm]
k	WSE wavenumber [rad/m]
k_{EB}	wavenumber for the Euler-Bernoulli beam theory [$\sqrt{\text{rad}}$/m]
$[\widehat{K}^g]$	global dynamic stiffness matrix
L	span between two end supports [m]
L^e	length of an element [m]
$M(x,t)$	bending moment [Nm]
n	number of sampling points
N	order of the Daubechies wavelet
N_x	constant axial pretension [N]
$Q(x,t)$	shear force [N]
r_N	tensile force-to-flexural rigidity ratio [1/m^2]
$\mathbf{\Gamma}^1$	first-order connection coefficient matrix (time domain)
$\mathbf{\Gamma}^2$	second-order connection coefficient matrix (time domain)
$i\gamma$	eigenvalues of $\mathbf{\Gamma}^1$ [rad/s]
$\mathbf{\Lambda}^1$	first-order connection coefficient matrix (wave domain)
$\mathbf{\Lambda}^2$	second-order connection coefficient matrix (wave domain)
$i\lambda$	eigenvalues of $\mathbf{\Lambda}^1$ [rad/s]
ρA	mass per length of beam [kg/m]
$\varphi(\tau-k)$	Daubechies scaling function at an arbitrary scale
Ω^1_{j-k}, Ω^2_{j-k}	connection coefficients

eigenvalue problem. The decoupled ODE's are then solved similarly as in SEM. In SEM, the time window is dependent on the value of damping and the dimensions of structure. It requires to be more wide for weakly damped and shorter dimension structures. A potential remedy to reduce these dependencies is to use artificial damping (AD) [15]. By using non-periodic boundary condition assumptions [19] for WSE, exactness of results could be free from those deficiencies previously noted such as lack of damping, structures having short dimensions, and small time windows. It should be noted that WSE method could also be used for analyzing undamped structures, where SEM could not work. This model can be used in time domain analysis without any transformations between different domains, something unlike SEM. Periodic boundary condition-based WSE formulation [19] can extract frequency dependent wave characteristics, like wavenumbers, directly. Moreover, [20,21] show the application of wavelet technique in FE domain.

Recent developments in research on axially moving structures were reviewed by Marynowski and Kapitaniak [22]. However, to the best of authors' knowledge, the WSE model has not yet been introduced in the literature for axially moving nanobeam structures and this should be the most remarkable innovation of this study. Thus, the objectives of this article are: (1) to develop a WSE model for axially moving EB pre-stressed beams based on nonlocal elasticity theory, (2) to highlight higher accuracy of this model as compared with those of classical FEM and SEM and (3) to investigate the effects of nanobeam properties, such as velocity, pretention and nonlocal parameter, on the vibration and wave characteristics, dispersion curves and instability.

2. Mathematical model

Based on the formulation derived in Appendix A, the governing equation for an element of axially moving EB pre-stressed beam based on nonlocal elasticity theory can be represented as

$$\begin{aligned}&\left[EI+(e_0a)^2(N_x-\rho Ac^2)\right]\frac{\partial^4 w}{\partial x^4}-2(e_0a)^2\rho Ac\frac{\partial^4 w}{\partial x^3\partial t}\\&-(e_0a)^2\rho A\frac{\partial^4 w}{\partial x^2\partial t^2}-(N_x-\rho Ac^2)\frac{\partial^2 w}{\partial x^2}\\&+2\rho Ac\frac{\partial^2 w}{\partial x\partial t}+\rho A\frac{\partial^2 w}{\partial t^2}=\left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]f(x,t).\end{aligned}\tag{1}$$

The force boundary conditions are given as

$$\begin{cases}Q(0,t)=-Q_{n\_1}(t)-N_x\frac{\partial w(0,t)}{\partial x}\\ M(0,t)=-M_{n\_1}(t)\\ Q(L^e,t)=Q_{n\_2}(t)-N_x\frac{\partial w(L^e,t)}{\partial x}\\ M(L^e,t)=M_{n\_2}(t)\end{cases},L^e:\ \text{the length of element.}\tag{2}$$

where $Q(x,t)$ and $M(x,t)$ are the shear force and bending moment define as

$$\begin{aligned}Q(x,t)\triangleq-\frac{\partial M}{\partial x}=&-\left[EI+(e_0a)^2(N_x-\rho Ac^2)\right]\frac{\partial^3 w}{\partial x^3}\\&-(e_0a)^2\left[-2\rho Ac\frac{\partial^3 w}{\partial x^2\partial t}-\rho A\frac{\partial^3 w}{\partial x\partial t^2}+\frac{\partial f}{\partial x}\right].\end{aligned}\tag{3}$$

$$\begin{aligned}M(x,t)\triangleq&\left[EI+(e_0a)^2(N_x-\rho Ac^2)\right]\frac{\partial^2 w}{\partial x^2}\\&+(e_0a)^2\left[-2\rho Ac\frac{\partial^2 w}{\partial x\partial t}-\rho A\frac{\partial^2 w}{\partial t^2}+f(x,t)\right].\end{aligned}\tag{4}$$

Fig. 1 shows an element of the nanobeam where $M_{n\_1}(t)$ and $Q_{n\_1}(t)$ are the bending moment and transverse shear force applied at $x=0$, and $M_{n\_2}(t)$ and $Q_{n\_2}(t)$ are the bending moment and transverse shear force applied at $x=L^e$.

3. Temporal discretization

The scaling transform of functions $w(x,t)$ and $f(x,t)$ can be done by a sequence of Daubechies scaling function $\varphi_{m,\mathcal{H}}(t)$ at an arbitrary scale m, as

$$w(x,t)=2^{m/2}W(x,\tau),\quad W(x,\tau)\triangleq\sum_{\mathcal{H}}w_{\mathcal{H}}(x)\varphi(\tau-\mathcal{H}),\quad \mathcal{H}\in\mathbb{Z}\tag{5}$$

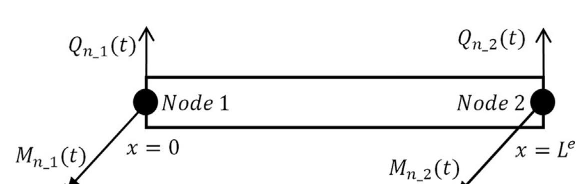

Fig. 1. A finite element of axially moving pre-stressed nanobeam.