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End condition effect on initial buckling performance of thin plates resting on tensionless elastic or rigid foundations

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ABSTRACT

It is well accepted that the non-contact buckling response of long rectangular plates subjected to compressive or shearing loadings is solely dependent on material and section properties, as well as boundary conditions along the two unloaded side edges. In addition, the boundary conditions at the loaded ends do not influence the critical load. The above rule was also applied to the analysis and design of plates in unilateral contact with elastic or rigid mediums although without any reasonable verification. The results contained in this paper reveal that the boundary condition at the loaded edges (end conditions) significantly affects the contact buckling performance. To investigate the end condition effect, a transfer function method is employed herein to study the initial buckling behaviour of thin plates with different boundary conditions resting on tensionless elastic or rigid foundations. For very long plates with both ends clamped, the buckling mode can be considered as a series of periodically repeating buckling waves and an infinite plate model with only one buckle wave being effective to predict the buckling behaviour. However, for long plates with simply supported loaded ends, local effects dominate the buckling mode and the buckling loads can be reduced by up to 18% compared to long plates with clamped loaded ends. Experimental tests and ABAQUS modelling also verify that the critical stresses for plates with simply supported loaded ends are lower than those for plates with clamped loaded ended plates.

1. Introduction

Plate element buckling is one of the controlling criteria in steel structural design.

The critical stress for compressive plate buckling can be expressed as [1]

$$\sigma_{cr} = \frac{K_{cr}\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \tag{1}$$

where σ_{cr} is the critical stress; E, ν, t, b are the elastic modulus, Poisson's ratio, thickness and width of the plate; K_{cr} is the buckling coefficient or critical load coefficient depending on the loading type, boundary conditions and plate aspect ratio, i.e., the ratio of plate length over plate width.

In general, for a long plate subject to compressive or shearing loads, the buckling coefficient tends to be a constant value which is solely determined by the boundary conditions at the two unloaded side edges. An example is the compressive buckling of laterally clamped plates with different boundary conditions at the loaded edges [1], where all theoretical buckling coefficients converge to a constant value of 6.97 (Fig. 1). In this figure, C and S refer to clamped and simply supported boundary conditions, respectively. The constraints at the loaded ends do not have any influence on the critical stresses of a long plate, and this is the case for long plates. As a result, Australian standards related to steel structure design [2,3] only considers lateral boundary conditions. The above rule was also applied to the analysis and design of plates in unilateral contact with elastic or rigid mediums although without any verification.

Skin buckling is a different plate buckling phenomenon that occurs in composite members [4–9]. It is related to delamination buckling of laminated plates where the material anisotropy may lead to contact between the buckled layer and the base medium (i.e. filler materials) [10–12]. In this case, plates separate either away from the filler material or remain in contact with the filler material. The phenomenon is also known as a contact buckling problem or a unilaterally constrained buckling problem. The contact buckling problem is usually modelled through a plate resting on a tensionless foundation that can be produced from a filler material. The foundation can range from elastic to rigid in behaviour. It is a challenging problem to

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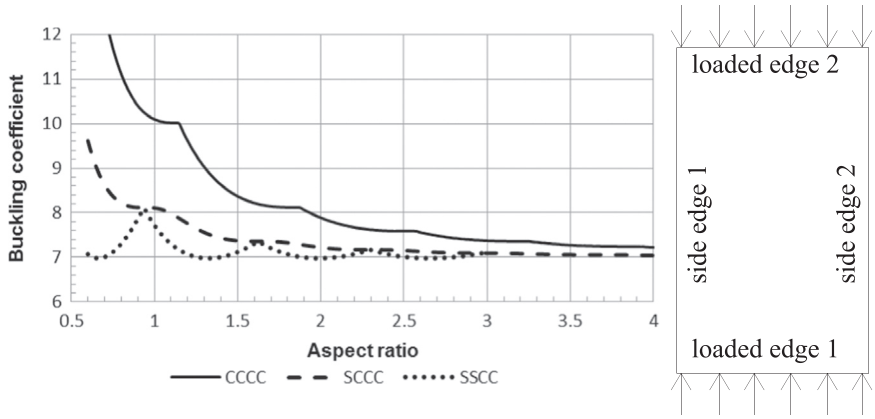

Fig. 1. Buckling coefficient for plates with different boundary conditions. (SSCC- simply supported on loaded edges and clamped on side edges; SCCC-simply supported on one loaded edge and clamped on the remaining edges; CCCC-clamped on four edges).

analyse due to nonlinearities arising from the unilateral constraints and the complexity of contact effects.

The earliest contact buckling model appeared in the 1950s [13], which was further extended to the initial compressive buckling [14–17], post-local buckling [11,12,18–20] and shear buckling analysis [21,22] of plates on tensionless rigid or elastic foundations. More studies have recently been conducted on the unilateral restrained buckling behaviour of multi-plates [23], profiled sheets [24], composite plates [25], and cylindrical plates [26].

Although there are a number of published studies relating to the unilaterally constrained buckling problem, aspects of the associated phenomena remain to be explained. An example is the influence of the boundary conditions at the loaded plate ends. Generally for a non-contact buckling response, a long plate may be conveniently represented as an infinite plate (Fig. 1). Two questions, however, present themselves, namely (i) what is the length required to apply the infinite plate approximation, and (ii) would a plate with simply supported ends behave in the same way as a plate with clamped ends? This paper therefore concentrates on the end condition effect through the buckling analysis of finite length plates with various boundary conditions resting on an elastic foundation (Table 1).

Table 1
Plate types under investigation.

Plate type	Loaded edge 1	Loaded edge 2	Side edge 1	Side edge 2
CCCC	Clamped	Clamped	Clamed	Clamped
SCCC	Simply supported	Clamped	Clamed	Clamped
SSCC	Simply supported	Simply supported	Clamed	Clamped
CCSS	Clamed	Clamped	Simply supported	Simply supported
SCSS	Simply supported	Clamped	Simply supported	Simply supported
SSSS	Simply supported	Simply supported	Simply supported	Simply supported

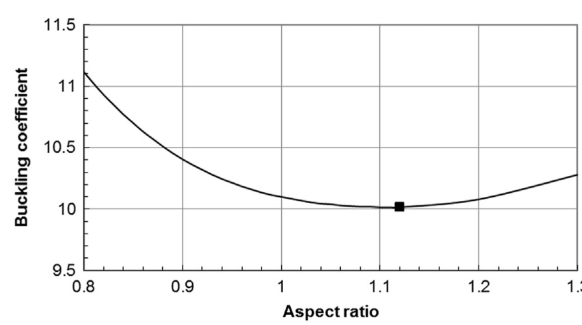

Fig. 2. Bukling coefficients for an unconstrained CCCC plate with varying aspect ratio.

2. Plates on rigid foundation

2.1. Modelling

A laterally clamped plate resting on a rigid, tensionless foundation is investigated as an example. The four edges conditions are CCCC, while the transverse displacement and all its differentials in the contact area between the plate and the rigid foundation are zero. The same conditions apply to the borderline between the noncontact zone and the contact zone. Thus, the boundary conditions for the buckle in the non-contact zone are the same as those for an unconstrained CCCC plate. This allows the use of an unconstrained CCCC plate model to simulate the contact plate model. Based on unconstrained plate buckling theory [1], the buckling coefficient for a non-contact CCCC plate can be obtained in terms of the aspect ratio of the buckle waves as shown in Fig. 2. The minimum value is $K_{cr}=10$ with aspect ratio $L/b=1.12$. It can therefore be determined that a rigidly constrained CCCC plate with $L/b\leq 1.12$ buckles into a one half-wave as an unconstrained plate. In addition, a longer plate with $L/b>1.12$ may buckle into one or several half-waves, of which each wave has the same profile as an unconstrained CCCC plate with $L/b=1.12$ and a critical load coefficient of 10. Since the position of the half-wave buckles are indeterminate (i.e. dependent on imperfections), the length and position of the contact region(s) in the plates are undefined.

Similarly, a laterally clamped plate (SCCC) with one loaded end simply supported and the other loaded end clamped, or a laterally clamped plate (SSCC) with both loaded ends simply supported, on a tensionless rigid foundation will buckle identically to an unconstrained SCCC plate for $L/b\leq 0.94$. For a longer plate with $L/b>0.94$, the plate will buckle into a half-wave close to the pinned end with the same profile as an unconstrained SCCC plate with $L/b=0.94$ and with other areas remaining in plane contact (Fig. 3). The buckling coefficient in this case is $K_{cr}=8.12$. The comparison of buckling coefficients in terms of varying aspect ratios among CCCC, SCCC and SSCC plates on rigid tensionless foundations are shown in Fig. 4.