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Modeling and analysis of nanobeams based on nonlocal-couple stress elasticity and surface energy theories

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ABSTRACT

This paper aims to develop a new non-classical Bernoulli–Euler model, taking into account the effects of a set of size dependent factors which ignored by the classical continuum mechanics. Among those factors are the microstructure local rotation, long-range interactions between a particle and the other particles of the continuum and the surface energy effects. The model used the modified couple-stress theory to study the effect of the local rotational degree of freedom of a specific particle. Furthermore, the surface elasticity model developed by Gurtin and Murdoch has been used to determine the surface energy effects on the behavior of the particle. The effects of the local rotation and surface energy are investigated in the framework of nonlocal elasticity theory, which is employed to study the nonlocal and long-range interactions between the particles. In addition, Poisson's effect incorporated in the newly developed beam model. The equations of equilibrium and complete boundary conditions of the new beam are derived using the principle of virtual work.

The developed model is validated, by comparing the obtained results with benchmark results. To illustrate the new model, analytical solutions for the static bending and critical buckling load are obtained. Numerical results reveal the significant effects of the nonlocal, microstructure, surface energy, length-to-height ratio and Poisson on the static bending and critical buckling load of nanobeams.

1. Introduction

It is clear that in spite of the power of classical local elasticity, in its domain of applications, it fails to describe many important size-dependent phenomena [5–7,18]. In the recent era of nanoscience and nanotechnology, many factors are responsible on the breakdown of classical continuum mechanics, among of them are the effects of microstructure degrees of freedoms, the long-range cohesive interaction and the surface energy. Many papers have been published to discuss the effect of each of these factors and the range of size dependency to consider each factor effect into account. Few publications have investigated the combined effects of more than one factor at the same time. The couple effects of nonlocal elasticity and surface properties on the static or dynamic response of nanostructures was studied by [26,4,42,41,16] and [17]. Effects of surface stress and microstructure on the response of micro/nanostructures were presented by [32,11,10,33,12,38]. However, no work includes the three effects at the same time and consequently the aim of this paper is to study the relative effects of all these three mentioned factors, simultaneously.

Voigt [37] assumed that the transfer of interaction between two neighborhood elements of a body is not only by means of force vectors but also by face-moment and body moment vectors. Rotation in continuum solid mechanics is divided into two kinds; one is independent called micro-polar rotation, which represents one sort of microstructure effects. The second is the anti-symmetric part of the displacement gradient field, called local rotation. In fact, the local rotation, at a point of the continuum, represents a constraint on the displacements at this point that induce an additional couple stress and, consequently, contributes on the strain energy density of the continuum. Gao and Chen [8] and Gao and Lin [9] have derived the constitutive equations of the nonlocal body moment associated with local rotation, based on the axiom of nonlocal continuum fields and nonlocal quasi-continuum theory.

From the other side, the nonlocal continuum theory considers the long-range interatomic cohesive force, but not considered as one of the microstructure effects. Consequently, it yields results which are dependent on the size of the body and similar to the classical continuum theory, where the lattice particle are taken as an idealized mass point [3]. It is not a theory for a continuum embedded with microstructure, but only for material involving

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long-range interaction. Assume a is the interatomic distance; c is the range of the cohesive force; and λ is the external characteristic length (such as the width of a crack or the thickness of a nano-layer). Thus, the size dependent range of application for the validity of the local elasticity theory which is based on the concept of short range cohesive force, is given by $a \ll c \ll \lambda$. But for the cases of $a \ll \lambda \ll c$, the local elasticity is no longer valid and nonlocal continuum should be applied [31].

The surface energy effect is the third factor responsible for the breakdown of the classical continuum mechanics. The surface is regarded as a membrane with a negligible thickness [14,15,43], where the atoms arrangements and material properties differs from those of the bulk material [44,45]. For a larger size, surface energy effects can be ignored because the ratio of the surface layer volume to the bulk volume is very small. However, for a higher ratio of surface layer volume to the bulk volume, such surface effect becomes effective [23].

In the present paper, a nonlocal couple-stress elastic continuum model taking into account the effect of surface energy is proposed. Eringen's nonlocal elasticity theory [6]; modified couple-stress theory [40] and surface elasticity theory developed by Gurtin and Shenoy [14,15], are exploited to develop an integrated model to investigate simultaneously the effects of long-range interatomic interactions, microstructure local rotation and surface energy effect. For the sake of clarification, details of the model are demonstrated while applying the steps of the model on Bernoulli–Euler nano-beam, as an example.

2. Theoretical formulations

Based on the Bernoulli–Euler hypothesis, all applied loads and geometry are such that the axial and lateral displacements u and w, respectively, of a point at a height z measured from the mid-plane and a distance x along the beam length in its deformed state, are assumed as to be functions of only x and z coordinates, such that, e.g., [34];

$$u(x,z) = z\phi(x) \cong -z\frac{dw(x)}{dx} \quad \text{and} \quad w(x,z) = w(x) \tag{1}$$

where $\phi(x)$ is the rotation angle of the centroidal axis of the beam.

2.1. The modified couple stress theory

According to the modified couple stress theory (MCST), [40], the constitutive equations are given by

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \tag{2}$$

$$m_{ij} = 2l^2\mu\chi_{ij} \tag{3}$$

where σ_{ij} and m_{ij} are, respectively, the components of the Cauchy stress and the deviatoric part of the couple stress tensor, λ and μ are Lame's constants in classical elasticity and δ_{ij} is the Kronecker delta. The material length scale parameter l measures the effect of couple stress [24,25]. Note that throughout the paper, the summation convention and standard index notation are used, with the Greek indices running from 1 to 2 and the Latin indices from 1 to 3 unless otherwise indicated.

The components of the infinitesimal strain ε_{ij} and the symmetric curvature tensor χ_{ij} are defined as [40]

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{4}$$

$$\chi_{ij} = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}) \tag{5}$$

with u_i being the displacement components and θ_i being the components of the rotation vector given by

$$\theta_i = \frac{1}{2}\varepsilon_{ijk}u_{k,j} \tag{6}$$

where ε_{ijk} is the permutation tensor.

From Eqs. (1) and (4)–(6), the following non-zero components of rotation vector, strain tensor and symmetric curvature tensor in the bulk of the current Bernoulli–Euler beam, are respectively, given by

$$\theta_y(t) = -\frac{dw(x,t)}{dx} \tag{7}$$

$$\varepsilon_{xx}(t) = -z\frac{d^2w(x,t)}{dx^2} \tag{8}$$

$$\chi_{xy}(t) = -\frac{1}{2}\frac{d^2w(x,t)}{dx^2} \tag{9}$$

Substituting Eq. (8) into Eq. (2) and Eq. (9) into Eq. (3), the following non-zero stress component are obtained

$$\sigma_{xx} = -z\tilde{E}\frac{d^2w(x,t)}{dx^2}, \quad \sigma_{yy} = \sigma_{zz} = -z\frac{\nu}{(1-\nu)}\tilde{E}\frac{d^2w(x,t)}{dx^2} \tag{10}$$

and

$$m_{xy} = -l^2\mu\frac{d^2w(x)}{dx^2} \tag{11}$$

The effective modulus of elasticity $\tilde{E}$ of the bulk can be defined as;

$$\tilde{E} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \tag{12}$$

where E and ν are, respectively, Young's modulus and Poisson's ratio of the beam material.

2.2. Surface elasticity theory

According to the surface elasticity theory [14,15], the surface layer of a bulk elastic material satisfies distinct constitutive equations involving surface elastic constants and surface residual stress. The governing equations for the surface layer of zero thickness as given by [14,15] are as follows

$$\tau_{\alpha\beta} = (\tau_s + [\lambda_s + \mu_s]u_{\gamma,\gamma})\delta_{\alpha\beta} + \mu_s\left(u_{\alpha,\beta} + u_{\beta,\alpha}\right) - \tau_s u_{\beta,\alpha} \tag{13a}$$

$$\tau_{n\alpha} = \tau_s u_{n,a} \tag{13b}$$

where α, β represent the in-plane Cartesian coordinates of the surface. μ_s and λ_s are the surface elastic constants and τ_s is the residual surface stress (i.e., the surface stress at zero strain). These three constants μ_s, λ_s and τ_s can be determined from atomistic simulations (e.g., [23]). $\tau_{n\alpha}$ is the out-of-plane components of the surface stress tensor.

From Eqs. (13) and (1), the non-zero components of the surface stresses are related to the displacement as follows

$$\tau_{xx} = \tau_s - z(2\mu_s + \lambda_s)\frac{d^2w}{dx^2} \quad \text{and} \quad \tau_{n\alpha} = \tau_s n_z\frac{dw}{dx} \tag{14a, b}$$

where n_z is the z-component of the unit outward normal vector $\mathbf{n}$ to the beam lateral surface.

2.3. Nonlocal differential constitutive relation

In the nonlocal elasticity theory, stress at a point in the continuum is a function not only of the strains at that point, but also of