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On the determination of the work hardening curve using the bulge test

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ABSTRACT

Hydraulic bulge test represents nowadays an important means to obtain higher accuracy on material characterization. One reason is the possibility of using the obtained biaxial stress–strain data to largely extend the hardening information extracted from the tensile test. The other reason is the use of biaxial data as input information when determining parameters for current most advanced yield criteria.

This contribution aims to obtain the material stress–strain hardening curve from the bulge test using a simpler experimental equipment, in which the output data is the hydraulic bulge pressure and the pole bulge height. This information is used to determine the sheet thickness and corresponding radius of curvature at the pole of the cap, which is the needed data to calculate the biaxial stress–strain curve, the stress being determined based on Laplace's equation from the membrane theory, a standard approach for this kind of analysis.

Analytical models are proposed relating the radius of curvature and the sheet thickness with the pole bulge height. These models are based in an extensive analysis of different material behaviors, which in turn are related to characteristic properties of sheet metals, as well as different geometries of bulge test. Geometric variables include bulge die diameter and the fillet radii located at the entrance of the die. The analytical formulas also include the material variables associated with the hardening behavior and the sheet anisotropy, with different interaction and weighting impact.

The extensive study also permits a deeper theoretical understanding of relations among the interconnecting variables and their influence on the accuracy of sheet thickness and radius of curvature determination, which directly influences the obtained biaxial stress–strain curve. This means, for example, the understanding between sheet thinning evolution or bulge curvature evolution during bulging and the corresponding relation with material plastic properties, hardening and anisotropy.

The validation of the methodology and the proposed analytical models is performed with experiments, both from developed experimental system and also from literature with different bulge geometric relations.

1. Introduction

Finite element analysis is currently essential in the design stage of sheet metal forming components. Its implementation, wide use and contribution to better predictability in the design of sheet metal forming processes depends on the accurate characterization of the material properties [1]. Sheet metal mechanical behavior is usually described using mathematical models, i.e. constitutive laws, and for each material, the parameters of such models are generally determined with resource to tensile and other monotonous strain path tests, such as shear test and biaxial tension.

The biaxial bulge test, under hydraulic pressure [2], can achieve relatively high strain values before necking, allowing the definition of the hardening law up to large plastic deformations. In the bulge test, the periphery of the metal sheet is restrained by a drawbead, which prevents its radial displacement. Hydraulic pressure is then applied to the metal blank, forming the sheet into a hemispherical geometry without using a punch, thus, minimizing any influence of contact with friction. The test conditions promote biaxial strain paths at the pole of the cap, which is perfectly spherical in a region close to the pole and inside a circle of constant latitude [3,4]. This test can be used to obtain the strain limits defining the points of Forming Limit Curves (FLC's) and to characterize the material hardening behavior. Normally the region under study is limited to the area around the pole of the cap, which using circular or elliptical dies will allow a wide variety of the principal strain ratios at the pole of the cap [5,6].

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The bulge test has been the subject of growing interest with a great relevance on the characterization of strain hardening laws of metal sheets. However, the identification of the parameters of the hardening law presents some difficulties [7]. Various methodologies can be considered and several approaches have been developed and adopted over the years [8]. When determining the strain hardening law of the material, using the membrane theory, it is necessary to record the hydraulic pressure and evaluate the radius of curvature and the sheet thickness at the pole, during the test. As example of using different methodologies, the determination of the sheet thickness can be performed based on strain measurements at the pole or using mathematical models, which take into account the geometry of the test [9,10]. Similarly, the radius of curvature at the pole of the cap can be determined using direct evaluation with different points at the pole or indirect evaluation with mathematical models using total height at the pole [8,10,11].

Regarding the direct evaluation, the use of optical measuring systems allows the description of the geometry and strain distributions on sheet surface during the bulge test. However, the evaluation of the stress *vs.* strain curves depends on assumptions and simplifications, whose assessment are still under study. For example, in a recent study Mulder et al. [12] examine the validity and the conditions for applying the membrane theory, which includes issues such as: radii of curvature evaluation, coordinated system for strain measuring to be used, equibiaxial stress state assumption in case of in-plane anisotropic materials, existence of bending stresses and through thickness stress due to the hydraulic pressure.

This paper presents a numerical study of the hydraulic bulge test with circular die, using the in-house finite element program *DD3IMP* [13,14]. It is assumed that the radii of curvature are evaluated at the mid-plane of the cap, the coordinated system for strain measuring is aligned with the orthogonal axes of symmetry of the sheet and the non-balanced biaxial stress state is determined with the corresponding strain ratio, which needs prior knowledge of the shape of the yield surface [12]. In this context, in-plane isotropic and anisotropic materials were described by Hill'48 criterion. The aim of the work is to propose an approach for analysing the results of the circular bulge test, in order to obtain the stress at the pole of the cap with simple equipment and not requiring further specific devices to determine the radius of curvature and thickness at the pole. The influence of the mechanical properties of the material as well as the geometry of the die on the evolution of the radius of curvature and the sheet thickness with the pole height is considered in this analysis.

2. Theoretical background

The analysis of the stress state near the pole of the metal sheet during the bulge test, using either circular or elliptical dies, can be performed with the aid of the membrane theory [15], as long as it is satisfied a small ratio between the sheet thickness and the bulge die, typically values lower than 1/50 [8,16]. Under these conditions, the bending stress can be neglected and assuming that the thickness stress σ_3 ($=\sigma_Z$) is zero, a relationship between principal stresses at the pole, the pressure and the geometry of the cap is given by:

$$\frac{\sigma_1}{\rho_1}+\frac{\sigma_2}{\rho_2}=\frac{p}{t}. \tag{1}$$

where σ_1 and σ_2 are the principal stresses in the sheet surface (assuming that the principal stress axes (O123) and anisotropy axes (O*xyz*) coincide), ρ_1 and ρ_2 are the radii of curvature, at half thickness, in the O*xz* and O*yz* planes, respectively, p is the hydraulic pressure and t is the sheet thickness.

In order to experimentally determine the strain hardening curve, the evolution of the following variables need to be obtained during the test: pressure, p, the radii of curvature, ρ_1 and ρ_2, and the sheet thickness at the pole, t. The thickness can be determined based on the knowledge of the initial thickness of the sheet, t_0, and the thickness strain, ε_3, through the following equation:

$$t=t_0\exp(-\varepsilon_3). \tag{2}$$

The strain, ε_3, can be obtained from the measurement of the principal strains in the sheet plane, ε_1 and ε_2, based on the condition of volume constancy during plastic deformation:

$$\varepsilon_3=-(\varepsilon_1+\varepsilon_2). \tag{3}$$

Since the radii of curvature are experimentally evaluated on the external surface of the cap, their correction should be done based on the following equation [17]:

$$\rho=\rho_{ext}-\frac{t}{2}, \tag{4}$$

where ρ is the radius of curvature at the half thickness of the cap, and ρ_{ext} is the radius of curvature of the external surface of the cap.

In the general case, i.e. anisotropic metal sheet, the membrane theory equation (Eq. (1)), contains two unknown variables, σ_1 and σ_2, which requires an additional equation for its determination. For metal sheets obeying to the Hill'48 criterion [18], this additional equation can be obtained by the plastic stress-strain relationships, assuming coincidence of the coordinate systems of principal stress (O123) and anisotropy (O*xyz*):

$$\begin{cases} d\varepsilon_1=d\lambda[H(\sigma_1-\sigma_2)+G(\sigma_1-\sigma_3)] \\ d\varepsilon_2=d\lambda[F(\sigma_2-\sigma_3)+H(\sigma_2-\sigma_1)] \end{cases}, \tag{5}$$

where F, G and H are the anisotropy parameters, $d\varepsilon_1$ and $d\varepsilon_2$ are increments of plastic deformation in the sheet plane, parallel to the O*x* and O*y* axes, respectively, and $d\lambda$ is a scalar factor of proportionality.

In the bulge test, it can be assumed that $\sigma_3=0$ and, based on Eq. (5), one can be written:

$$\frac{d\varepsilon_1}{d\varepsilon_2}=\frac{\sigma_1(G+H)-\sigma_2 H}{-\sigma_1 H+\sigma_2(F+H)}. \tag{6}$$

Consequently, Eqs. (1) and (6) allow determining the principal stresses, σ_1 and σ_2, in case of circular and elliptical dies, assuming that the parameters, F, G and H of the Hill'48 criterion are known.

To calculate the equivalent stress, $\overline{\sigma}$, and the equivalent strain, $\overline{\varepsilon}$, values that characterize the hardening behavior, the following equations can be used [6,18]:

$$\overline{\sigma}=\sqrt{(G+H)\sigma_1^2+(F+H)\sigma_2^2-2H\sigma_1\sigma_2}, \tag{7}$$

$$\overline{\varepsilon}=\sqrt{F\left[\frac{G\varepsilon_2-H\varepsilon_3}{FG+GH+HF}\right]^2+G\left[\frac{F\varepsilon_1-H\varepsilon_3}{FG+GH+HF}\right]^2+H\left[\frac{F\varepsilon_1-G\varepsilon_2}{FG+GH+HF}\right]^2}. \tag{8}$$

In case of isotropic materials, obeying the von Mises yield criterion, the principal stresses, σ_1 and σ_2, in bulge tests performed either with circular or elliptical dies, can be calculated using Eq. (1) and simplifying the Eq. (6), as follow:

$$\frac{d\varepsilon_1}{d\varepsilon_2}=\frac{\sigma_1-0.5\sigma_2}{-0.5\sigma_1+\sigma_2}. \tag{9}$$

Similarly, the equivalent stress and strain can be calculated using the following simplified equations (von Mises):

$$\overline{\sigma}=\sqrt{(\sigma_1)^2+(\sigma_2)^2-\sigma_1\sigma_2}, \tag{10}$$