International Journal of Mechanical Sciences 105 (2016) 273–278

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Theoretical prediction of temperature dependent yield strength for metallic materials

Weiguo Li [a,b,*], Xianhe Zhang [a], Haibo Kou [a], Ruzhuan Wang [a], Daining Fang [c]

[a] State Key Laboratory of Coal Mine Disaster Dynamics and Control (Chongqing University), College of Aerospace Engineering, Chongqing University, Chongqing 400030, China
[b] State Key Lab for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an 710049, China
[c] LTCS and College of Engineering, Peking University, Beijing 100871, China

ARTICLE INFO

Article history:
Received 9 July 2015
Received in revised form
24 October 2015
Accepted 11 November 2015
Available online 23 November 2015

Keywords:
Metallic material
Energy methods
Yield strength
Temperature dependent model

ABSTRACT

Based on a kind of equivalence between heat energy and distortional strain energy, it is assumed that there is a constant maximum which includes both the distortional strain energy and the corresponding equivalent heat energy associated with material yield. A temperature dependent yield strength model is then developed for metallic materials. The model establishes the quantitative relationship of the yield strength, temperature, elastic modulus, the specific heat capacity at constant pressure and Poisson's ratio. The comparisons between the model and experiments are made, and the agreement between theory and experiment is striking. It is noteworthy that this model has no fitting parameters. The strength of the model is its generality and ability to easily predict the temperature dependent yield strength at arbitrary temperatures.

1. Introduction

The yield strength is very sensitive to temperature. The work conditions of metallic materials are becoming more and more complicated. This leads to the high demand for the yield strength of metallic materials under high temperature or low temperature. In the past, a large quantity of experimental work has been reported about the temperature dependent yield strength of metallic materials [1–9]. But the yield strength tests of metallic materials at different temperatures are very inconvenient. At present, there are some yield strength theories for the different metallic materials considering the effect of temperature [10–12]. The current theoretical models are often obtained through fitting the experiment data. Those kinds of models are always suitable for one material. There are also some researchers using molecular dynamics simulation [13,14] to calculate the yield strength of metallic materials under different temperatures. Yet their computing objects are often at the nanoscale because of their large calculation amount. So it is very necessary and important to research and build a more universal temperature dependent yield strength model for metallic materials.

Dislocation slip is responsible for plastic slip in most crystalline materials [15]. If strain is applied to a metallic specimen at high temperature, the resistance of lattice atom to dislocation slip will decrease, and the dislocation will move faster. Similarly if some strain is applied, grain boundary premelting will happen at a low temperature which is much lower than melting point [16]. It is not difficult to find that temperature rising and stress increasing have some kind of equivalent relationship for the yielding of material. Celia Reina and Jaime Marian also mentioned that thermal strain fluctuations will carry the dislocation system over the barrier, and trigger the yielding of materials [17]. Yielding of plastic materials will occur once accumulated energy reaches a critical energy of the crystal bonding force [18]. It can be observed that both the deformation energy and the heat energy contribute to the onset of yielding of materials. The von Mises yield criterion states that yielding of materials occurs when the elastic deformation energy per unit volume reaches a certain value, no matter under what kind of stress state without regard to temperature. Learning from the previous work of our group which is a temperature dependent fracture strength model developed from an energy viewpoint, a new way was found to take into account the temperature effect on the yielding of metallic materials [19]. In the previous work, it is assumed that there is a maximum constant value for the maximum storage of energy in unit volume, which is associated with the onset of material fracture failure and can be interpreted as a critical fracture failure including the strain energy and the corresponding equivalent heat energy. In this work, we assume that

* Corresponding author at: Chongqing Key Laboratory of Heterogeneous Material Mechanics, College of Aerospace Engineering, Chongqing University, Chongqing 400030, China. Tel.: +86 13452029896; fax: +86 23 65102421.
E-mail address: wgli@cqu.edu.cn (W. Li).

http://dx.doi.org/10.1016/j.ijmecsci.2015.11.017
0020-7403/© 2015 Elsevier Ltd. All rights reserved.

yielding of materials at temperature T occurs when the sum of the elastic deformation energy per unit volume and the corresponding heat energy reaches a certain value. It is assumed that the elastic deformation energy and the corresponding heat energy at temperature T have some kind of equivalent relations according to their contribution to the onset of yielding of materials.

A new idea of modeling based on a kind of equivalence between heat energy and distortional strain energy is proposed. A new temperature dependent yield strength model is established. The model develops a quantitative relationship of the yield strength, temperature, elastic modulus, the specific heat at constant pressure and Poisson's ratio.

2. Theoretical derivations

Based on the above assumptions, the critical yield energy density has the form:

$$W_{\text{total}} = W_d(T) + K W_T(T), \tag{1}$$

where W_{total} is the maximum constant value for the maximum storage of energy associated with the onset of material yielding, which is dependent on the kind of material and its microstructure. $W_d(T)$ is the elastic deformation energy per unit volume associated with materials yield at temperature T. Here we consider uniaxial tension situation, because unidirectional static tensile test is the most widely used test method of mechanical properties in the industrial and materials science. In the case of uniaxial tension, the elastic deformation energy per unit volume can be expressed as:

$$W_d(T) = \frac{1+\mu_T}{3E_T}(\sigma_y(T))^2, \tag{2}$$

where $\sigma_y(T)$, μ_T and E_T are the yield strength, Poisson's ratio and Young's modulus respectively at temperature T.

$W_T(T)$ is the heat energy per unit volume at temperature T, which can be expressed as:

$$W_T(T) = \int_0^T \rho C_p(T) dT, \tag{3}$$

where $C_p(T)$ is the specific heat capacity for constant pressure and temperature T; ρ is the density (here, ρ is considered as a constant because of its weak temperature dependence); T is the temperature (in Celsius).

K, assumed constant, is the dimensionless ratio coefficient between the elastic deformation energy and the heat energy. Because the elastic deformation energy and the heat energy are not completely equivalent. K is used to reflect their transformational relation.

When $T = T_m$, the melting point of the material, the liquid state can no longer be subjected to any stretching strain. Thus $\sigma_y(T_m) = 0$ and there is no contribution from applied work:

$$W_d(T_m) = 0, \tag{4}$$

Substituting Eq. (4), $T = T_{\mathrm{m}}$ into Eq. (1) we can conclude that:

$$W_{\text{total}} = K \int_0^{T_m} \rho C_p(T) dT, \tag{5}$$

Substituting $T = T_0$ into Eq. (1) we can obtain that:

$$W_{\text{total}} = \frac{1+\mu_{T_0}}{3E_{T_0}}(\sigma_y(T_0))^2 + K \int_0^{T_0} \rho C_p(T) dT, \tag{6}$$

where T_0 is an arbitrarily reference temperature.

The combination of Eqs. (5) and (6) yields

$$K = \frac{1+\mu_{T_0}}{3E_{T_0}}(\sigma_y(T_0))^2 \Big/ \int_{T_0}^{T_m} \rho C_p(T) dT, \tag{7}$$

Substituting K (Eq. (7)) into Eq. (1) yields

$$W_{\text{total}} = \frac{1+\mu_T}{3E_T}(\sigma_y(T))^2 + \frac{1+\mu_{T_0}}{3E_{T_0}}(\sigma_y(T_0))^2 \int_0^T C_p(T) dT \Big/ \int_{T_0}^{T_m} C_p(T) dT, \tag{8}$$

Substituting Eq. (5) into Eq. (8), $\sigma_{\mathrm{y}}(T)$ can be expressed as:

$$\sigma_y(T) = \left[\frac{(1+\mu_{T_0})E_T}{(1+\mu_T)E_{T_0}} \left(1 - \frac{\int_{T_0}^{T} C_p(T) dT}{\int_{T_0}^{T_m} C_p(T) dT} \right) \right]^{0.5} \sigma_y(T_0), \tag{9}$$

The specific heat capacity $C_p(T)$ can easily be found in material handbook and the temperature dependent Young's modulus can more easily be obtained by experiments compared to the yield strength. Therefore the temperature dependent yield strength of metallic materials at arbitrary temperatures can be predicted without any fitting parameters.

3. Experimental validation

Using the proposed temperature dependent yield strength model, the temperature dependent yield strength of some metallic materials were predicted. The comparisons between the calculated results and the experimental data are made. In the calculations, the used specific heat capacities of materials are obtained from the material handbooks [20,21]. In this work, the effect of temperature dependent Poisson's ratio on the yield strength is neglected. And we do not adjust T_0 to achieve the agreement between theory and experiment.

3.1. Q345 (16Mn)

The values of variables in Eq. (9) used to obtain the predictions are shown (see below). The lattice structure of Q345 (16Mn) at the researched temperature section is mainly body centered cubic structure (BCC). Table 1 shows the Young's modulus used to obtain the predictions [22]. Other parameters used to obtain the predictions:

$\sigma_0 = 345$ MPa; melting point: $T_{\mathrm{m}} = 1809$ K; iron's specific heat capacities are used here, the concrete form is shown as follows

Table 1
Young's modulus of Q345 (16Mn) used to obtain the predictions.

Temperature/°C	Young's modulus/MPa
20	203000
100	200361
150	197899.6
200	194068
250	188409.4
300	180467
350	169784.1
400	155904
450	138369.9
500	116725
550	90512.7
600	59276.1