International Journal of Mechanical Sciences 105 (2016) 291–303

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Dynamical behavior of a double-beam system interconnected by a viscoelastic layer

Y.X. Li [a], Z.J. Hu [a,b], L.Z. Sun [a,*]

[a] Department of Civil and Environmental Engineering, University of California, Irvine, CA 92697-2175, USA
[b] School of Transportation, Wuhan University of Technology, Wuhan 430070, PR China

ARTICLE INFO

Article history:
Received 30 June 2015
Received in revised form
29 October 2015
Accepted 14 November 2015
Available online 22 November 2015

Keywords:
Double-beam system
Viscoelastic layer
Winkler layer
Semi-analytical method
Modal expansion method
Orthogonality condition

ABSTRACT

As an idealized model of modern beam-type structures, the double-beam system has been specified in recent decades. While there have been various research efforts on the double-beam system, most of them are over-simplified, among which the viscoelastic damping mechanism of interlayer is often neglected. This paper presents a semi-analytical method to investigate the natural frequencies and mode shapes of a double-beam system interconnected by a viscoelastic layer. The two beams can be with different beam-masses, beam flexural rigidities and boundary conditions, as well as with and without the Winkler layer below lower beam, indicating that there is no restriction or assumptions on beams connected with the viscoelastic layer damping. The modal-expansion iterated method is further applied to determine the forced vibration responses in the double-beam system based on the natural frequencies and mode shapes obtained from the free-vibration analysis. A specific orthogonality condition for the double-beam system is derived, and then applied to decouple the differential equations of motion. Numerical examples are demonstrated and discussed in details to verify the efficiency of the proposed methodology, which can further help characterize the dynamic responses and design work for double-beam structures.

1. Introduction

Beam-type structures are an essential element in structural systems and are widely used in engineering disciplines, especially in civil, aerospace and mechanical engineering. Corresponding research efforts on the dynamic behavior and vibration suppression on beam-type structures attract a great deal of attention from researchers and engineers. All investigations aim to find what exactly the characteristics of beam-type structural vibration is, and then, indicate a means to reduce or control the vibration into an accepted level. On one hand, single-beam structures, the one-dimensional continuous systems with various boundary conditions and different excitations, have been studied extensively due to its simplicity. On the other hand, double-beam systems, consisting of two one-dimensional continuous beams connected by a uniformly distributed viscoelastic layer, have been explored in the past decade due to their unique design for modern engineering applications such as sandwich or composite beams, continuous dynamic vibration absorbers, and active constrained layer damping.

Most research efforts in the literature have simplified the double-beam systems as two identical beams with simply supported boundary conditions. Among those, the viscoelastic damping characteristics of the connecting layer between the two beams have been ignored [1–14]. To take into account the damping effect, researchers frequently make some assumptions and simplifications. For example, based on their early work on the axially-loaded damped Timoshenko beam on a viscoelastic foundation [15], Chen and Sheu [16,17] studied the free vibration, dynamic response and static buckling of two identical beams with a viscoelastic material layer in between. Li and Hua [18] introduced a finite-element method for a double-beam system which can have unequal masses, unequal flexural rigidities and arbitrary boundary conditions. However, to take into account the damping effect, they assumed the two beams must be identical. Kessel and Raske [19] solved a double-beam system under the cyclic moving load with both individual damping and relative damping. While the two beam components can be different, they must have same simply supported boundary conditions. Abu-Hilal [20] investigated the dynamic response of a double-beam system with viscoelastic layer damping traversed by a constant moving load and obtained the dynamic deflections of both beams in analytical closed forms. By using the direct Lyapunov method and simplifying the damping as viscous damping of each beam itself, Pavlovic et al. [21] investigated the stability and instability of a double-beam system subjected to compressive axial loading. In those two papers, the two beams are identical with same simply supported boundary condition. Vu et al. [22]

* Corresponding author. Tel.: +1 949 824 8670.
E-mail address: lsun@uci.edu (L.Z. Sun).

http://dx.doi.org/10.1016/j.ijmecsci.2015.11.023
0020-7403/

presented an exact method to analyze a two-beam system with a viscoelastic layer, with boundary conditions on the same side of the system being same and the two identical beams. Irie et al. [23] discussed the steady-state responses of a double-beam system under the sinusoidal force and a transfer matrix technique is adopted for solving the differential equations, but the damping considered in it is beam internal damping instead of the viscoelastic layer damping. Cottle [24] explored the layered beam with mixed boundary conditions and a semi-analytical method was applied to solve the equations with the assumption of same lateral displacements in beams. Xin and Gao [25] applied the double-beam system into a specific engineering structure, a bridge with a viscoelastic layer and a slab track on it, and used the finite element method and multibody dynamics theory to solve the problem. Dublin and Friedrich [26] obtained the forced vibration responses for two elastic beams interconnected by spring-damper system, with two spring-damper systems between two beams instead of uniformly distributed spring-damper systems. Oniszczuk [27] studied the analytical solutions for the damped transverse vibration of a simply-supported double-string system which has two identical viscoelastic strings and a viscoelastic layer between them. Oniszczuk's model was later adopted by Wu and Gao [28] to obtain the responses of a simply supported viscously damped double-beam system under moving harmonic loads. Other similar structures have also been analyzed, such as sandwich beams [29–33], continuous dynamic vibration absorbers [34–38], and composite layered foundations [39].

While there have been plenty of research efforts investigating the double-beam systems as shown above, most of them treat the viscoelastic layer damping as zero. Some limited efforts consider the damping effect with simplified double-beam systems. In real engineering practices, such as floating slab track on bridge, robotic arm in space station and two-stage vibration isolation system for precision instrument, the damping is an inherent properties of the materials for viscoelastic layer and its value cannot be ignored and the structure cannot be always simplified as above. Therefore, a general double-beam system with arbitrary viscoelastic layer damping must be considered. This paper presents a semi-analytical method to obtain the natural frequencies and corresponding mode shapes for a general double-beam system, in which the viscoelastic layer damping is nonzero and two beams may have Winkler layer below the lower beam, unequal masses, unequal flexural rigidities and arbitrary boundary conditions. In addition, to the double-beam system with viscoelastic layer and the Winkler layer, the forced vibration excited by arbitrary loading is analyzed using the classical modal expansion method and a proposed iteration method, based on the natural frequencies and mode shapes obtained from the free vibration analysis. A specific orthogonality condition for that double-beam system is derived and applied to decouple differential equations. The natural frequencies and mode shapes are calculated by the semi-analytical method for six cases of arbitrary masses, arbitrary flexural rigidities and arbitrary boundary conditions models. Furthermore, various double-beam system models are studied with a concentrated harmonic force in the midspan of upper beam to conduct the systematic parametric analysis of the structural resonance condition and dynamic responses.

2. Formulation of the problem

As shown in Fig. 1, the physical model of a double-beam system includes an upper beam and a lower beam joined by a uniformly distributed-connecting viscoelastic layer and with a Winkler layer below the lower beam. While homogeneous and prismatic, both beams can have different masses, flexural rigidities, and boundary conditions. The force-equilibrium equations are obtained based on the differential elements of both beams and the forces (Fig. 2) as:

$$-V_1(x,t)+K(W_1-W_2)+C\left(\frac{\partial W_1}{\partial t}-\frac{\partial W_2}{\partial t}\right)+f_{I1}(x,t) -f_1(x,t)+\left[V_1(x,t)+\frac{\partial V_1(x,t)}{\partial x}dx\right]=0 \tag{1a}$$

$$M_1(x,t)+V_1(x,t)dx=M_1(x,t)+\frac{\partial M_1(x,t)}{\partial x}dx \tag{1b}$$

$$-V_2(x,t)-K(W_1-W_2)-C\left(\frac{\partial W_1}{\partial t}-\frac{\partial W_2}{\partial t}\right)+f_{I2}(x,t)-f_2(x,t) +K_W W_2+\left[V_2(x,t)+\frac{\partial V_2(x,t)}{\partial x}dx\right]=0 \tag{1c}$$

$$M_2(x,t)+V_2(x,t)dx=M_2(x,t)+\frac{\partial M_2(x,t)}{\partial x}dx \tag{1d}$$

where $W_i(x,t)$, $V_i(x,t)$, and $M_i(x,t)$ are the transverse deflections, shear forces, and bending moments in the beams, respectively. Further, $f_{Ii}(x,t)$ is the inertia force of each differential beam element with x and t being the spatial co-ordinate and the time and $i=1$ or 2 representing the upper beam (1) or lower beam (2). The material constants K, C, and K_W are the stiffness, damping coefficient of the viscoelastic layer, and the stiffness of the Winkler layer, respectively. In addition, $f_1(x,t)$ and $f_2(x,t)$ are the exciting forces acting on the upper and lower beams, respectively.

Substituting the bending moment $M_i(x,t)=E_iI_i\frac{\partial^2 W_i}{\partial x^2}$ and the inertia force $f_{Ii}(x,t)=\rho_iA_i\frac{\partial^2 W_i}{\partial t^2}$ into Eq. (1), and considering the two beams as uniform and homogeneous which can be denoted as $E_i I_i=e_i$ and $\rho_iA_i=\overline{m}_i$, the governing equations of motion of the double-beam system (Fig. 1) can be derived as:

$$e_1\frac{\partial^4 W_1}{\partial x^4}+K(W_1-W_2)+C\left(\frac{\partial W_1}{\partial t}-\frac{\partial W_2}{\partial t}\right)+\overline{m}_1\frac{\partial^2 W_1}{\partial t^2}=f_1(x,t) \tag{2a}$$

$$e_2\frac{\partial^4 W_2}{\partial x^4}-K(W_1-W_2)-C\left(\frac{\partial W_1}{\partial t}-\frac{\partial W_2}{\partial t}\right)+K_W W_2+\overline{m}_2\frac{\partial^2 W_2}{\partial t^2} =f_2(x,t) \tag{2b}$$

The initial conditions in general form are as follows:

$$W_1(x,0)=W_{10}(x),\ W_2(x,0)=W_{20}(x),\ \dot W_1(x,0)=V_{10}(x),\ \dot W_2(x,0) =V_{20}(x) \tag{3}$$

The commonly used boundary conditions at the ends ($x=0, L$) are listed as follows:

$$\text{Simply supported}: W_i(0,t)=W_i(L,t)=W_i''(0,t)=W_i''(L,t)=0 \tag{4a}$$

$$\text{Clamped}: W_i(0,t)=W_i(L,t)=W_i'(0,t)=W_i'(L,t)=0 \tag{4b}$$

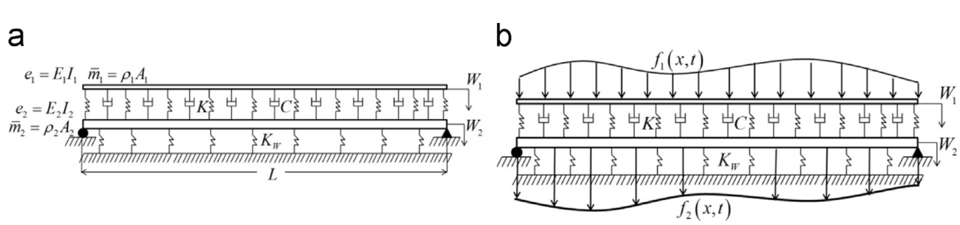

Fig. 1. The physical model of a double-beam system; (a) free vibration model; (b) forced vibration model.