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An advanced model for the numerical analysis of the radial stress in center-wound rolls

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ABSTRACT

In the case of roll-to-roll systems, winding is an important process and determines the quality of the final products. During the winding process, the winding tension determines the distribution of stress in the radial direction, i.e., the radial stress in the wound rolls. In order to optimize the winding tension, it is essential to have a model that can estimate the radial stress caused by the tension. However, to the best of our knowledge, no radial stress model that considers the effects of gravitation and the bending stress in the wound roll has yet been reported. In this study, we developed an advanced radial stress model that considers the effects of both these parameters on the radial stress. The accuracy of the developed model was verified experimentally for wound rolls of various materials. Finally, the effects of gravitation, the bending stress, and the winding tension on the radial stress were analyzed using the model. The model was found to be useful for analyzing the tension induced during the winding of several materials. Furthermore, the results of the performed analysis provided insights regarding how the radial stress is affected by the characteristics of the material being wound and the winding tension.

1. Introduction

Winding is the final process in the manufacturing of roll-to-roll systems, which are used to produce strip steel, fibers, plastics, and printed materials [1–9]. A center-wound roll is the most compact and efficient form of storing a wound-up material [10–12]. However, in such rolls, defects such as starring, buckling, and air gaps between the wound layers can occur because of the irregular distribution of the internal stress [13–15]. As the distribution of the internal stress in a wound roll is determined by the winding tension, determining the optimal winding tension while taking into account the characteristics of the winding material is essential. A taper tension profile can be used to represent the variations that occur in the winding tension with the winding radius. The profile can be determined by using taper tension models that are functions of the winding radius [13,16] or by controlling the winding tension such that it increases or decreases as the winding length increases [17]. Two types of representative taper tension profiles are used in industrial winding processes: linear and hyperbolic. In the linear taper tension profile (i.e., the linear profile), the winding tension decreases steadily and linearly, whereas in the hyperbolic taper tension profile (i.e., the hyperbolic profile), it decreases parabolically [18]. Taper profiles consist of a radius ratio, which is the ratio of the radius of the wound roll to that of the core, and a taper value, which determines the slope of the profiles. A number of researches have attempted to optimize the existing taper profiles and to develop novel optimized ones for the radial stress distribution [13,19,20]. From the results of these studies, it can be inferred that a mathematical model that can be used to analyze the distribution of the radial stress caused by the winding tension is necessary for determining the optimal taper profile. A model-based analysis of the stresses in wound rolls was first performed by Altmann [21]. He developed a radial stress model for an anisotropic, linear, and elastic wound material as a function of the winding tension. Yagoda developed a closed-form solution for the radial stress and confirmed that the core compliance could be used an inner boundary condition [22]. Hakiel extended the findings of these studies by considering the nonlinear behavior of wound rolls. He incorporated nonlinear material properties into the numerical solutions for the stresses in wound rolls [23]. Good et al. modified the boundary conditions related to the stress in the outer hoop in Hakiel's model and achieved better estimates [24]. Furthermore, Mollamahmutoglu and Good developed a large-deformation winding model based on the large deformation theory to analyze highly extensible and compressible materials such as nonwoven fabrics [25]. Burns et al. developed a

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radial stress model that took into account the residual stress in a wound roll [26]. The residual stress depends on the changes that occur in the winding tension with the radius of the wound roll. Li et al. developed a radial stress model that incorporated the effects of gravity in a steel coil [27]. This model was based on Benson's model [19]; therefore, it did not take into account the effects of the winding and bending stresses on the radial and residual stresses. A review of previous studies indicates that almost all previously proposed radial stress models are applicable only to materials with low densities and bending stiffnesses. Therefore, conventional models are not suitable for analyzing the radial stresses in wound rolls with large weights and high bending stiffnesses, with the bending stiffness being a function of the elastic modulus and the area moment of inertia.

In this study, an advanced radial stress model was developed on the basis of Burns' model while taking into account the density of the wound material, the bending stress, and the residual stress. In the developed model, the effects of the density and the bending stress were reflected in terms of the radial stress caused by the gravitational effect and the residual stress. Furthermore, the estimation accuracy of the developed model was determined experimentally for various plastic substrates having different densities and elastic moduli, such as polypropylene (PP), polyethylene terephthalate (PET), and polyimide (PI). The radial stress was measured using force-sensing resistor (FSR) sensors. The experimental results showed that the developed model could predict the behavior of the radial stress in the tested materials with high accuracy. Finally, the effects of the density, bending stiffness, and winding tension on the radial stress were analyzed using the developed model. The simulation results showed that the maximum radial stress increased with an increase in the density of the wound material and a decrease in the bending stress. Further, the maximum radial stress also increased with an increase in the winding tension; however, this tendency of the radial stress by increase of the radius ratio can be changed by the effect of the radial stress and the residual stress due to the bending stress. The developed model can be used to estimate the radial stress and optimize the winding tension for various winding conditions and various wound materials. Furthermore, the simulation results obtained using the developed model provided insights into the radial stress behavior of wound materials in terms of the material characteristics and winding conditions.

2. Development of an advanced radial stress model

2.1. Assumption and boundary conditions

To develop the radial stress model, the effects of the hoop stress, residual stress, and bending stress on the radial stress were identified. The following aspects were considered in order to obtain the equations to represent the relationships between the various internal stresses:

1. The correlation between hoop and radial strains in the wound roll.
2. The force balance equation in an arbitrary segment of the wound layer.
3. The residual stresses in an arbitrary wound layer.

It was assumed that the radial stress develops under the following conditions [21,24,26]:

1. The radial and hoop deformations in the wound material are below the elastic limit.
2. The radial stresses and strains are the principle components.
3. The gap between the wound layers is negligibly small. Further, a slip does not exist between the adjacent wound layers.
4. Stress is not generated along the width of the wound roll (i.e., the stress along the *z*-axis is zero).
5. The gravitational force is a function of the density of the wound material, the acceleration due to gravity, and the angle between the direction of the radial stress and that of the gravitational force acting on the wound roll.
6. The maximum compressive stress caused by the bending stress is the stress acting on the wound roll.

In this study, assumptions 5 and 6 were adopted to incorporate the effects of the density and the bending stress in the developed model. Assumption 5 considers the effects of the gravitational force in each wound layer on the radial stress. Assumption 6 states that the maximum compressive stress generated in the bottom surface of a wound layer affects the radial stress transferred to the adjacent wound layer. To solve the second-order differential equation for the radial stresses obtained from the relationships equations, the following two boundary conditions were adopted [26,28]: the radial stress in the outermost layer is zero and the radial stress and strain in the core are the same as those in the layer next to the core.

2.2. Correlation between the hoop and radial strains in the wound roll

Fig. 1(a) shows a schematic of a center-wound roll and (b) the deformation of the wound layers caused by the radial stress. In Fig. 1(b), the region enclosed by the bold lines and the gray region represent the wound layers before and after radial deformation, respectively. The radial displacement caused by the deformation is defined as $u(r)$. Consequently, the radial displacement, u, is defined as a function of the radius of the wound roll (r). The radial displacement in the first layer, which is next to the core, is defined as $u|_{r=c}$ $(=u(r)|_{r=c})$; then, the sum of the radial displacements from the first layer to the second layer can be obtained as follows:

$$u|_{r=c+\mathrm{th}} = u\Big|_{r=c} + \left(\frac{\mathrm{d}}{\mathrm{d}r}u\,\mathrm{d}r\right)\Big|_{r=c} + \alpha \tag{1}$$

where α is the gap between first and second layers. The subscripts c and th represent the radius of the core and the thickness of the wound material, respectively.

Based on assumption 3, (1) can be simplified as

$$u|_{r=c+\mathrm{th}} = u\Big|_{r=c} + \left(\frac{\mathrm{d}}{\mathrm{d}r}u\,\mathrm{d}r\right)\Big|_{r=c} \tag{2}$$

This equation can be generalized for the sum of the radial displacements from the first layer to the nth layer as follows:

$$u|_{r=r+\mathrm{th}} = u + \frac{\mathrm{d}u}{\mathrm{d}r}\mathrm{d}r \quad (u = u(r)) \tag{3}$$

The radial displacement in the nth layer can be obtained as

$$u|_{r=r+\mathrm{th}} - u = \frac{\mathrm{d}u}{\mathrm{d}r}\mathrm{d}r \tag{4}$$

The radial strain can be derived by differentiating the corresponding radial displacement and is the sum of the radial strains caused by the hoop and residual stresses.

$$\epsilon_{\mathrm{rr,total}} = \epsilon_{\mathrm{rr}} + \epsilon^*_{\mathrm{rr}} = \frac{1}{\mathrm{d}r}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\mathrm{d}r\right) + \frac{1}{\mathrm{d}r}\left(\frac{\mathrm{d}u^*}{\mathrm{d}r}\mathrm{d}r\right) = \frac{\mathrm{d}u}{\mathrm{d}r} + \frac{\mathrm{d}u^*}{\mathrm{d}r} \tag{5}$$

where $\varepsilon_{\mathrm{rr}}$ and $\varepsilon^*_{\mathrm{rr}}$ are the strain in the radial direction and its corresponding residual strain, respectively. Further, u^* is the displacement in the radial direction caused by the residual stress.

The sum of the hoop displacement and the strain can be obtained from (6) and (7). The strain is the sum of the hoop and