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Post-buckling modeling for strips under tension and residual stresses using asymptotic numerical method

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ABSTRACT

In many industrial processes, geometric strip defects may be generated by buckling due to excessive residual stresses and these defects are difficult to control. Usually residual stresses have complex distributions so that defects with complex shapes appear. In strip rolling, strips are fabricated under tension, which has an impact on the shape and the amplitude of the defects. This tension can hide completely or partially the defects that increase and evolve during tension release. In this work we calculate flatness defects of strips generated by residual stresses, with and without tension, by using a shell finite element model based on the asymptotic numerical method (ANM).

1. Introduction

In thin structures, buckling phenomena are usually associated with collapse of structures. In the load–displacement response, buckling loads correspond to bifurcation or limit points (see Fig. 1). In many manufacturing processes, buckling induces geometrical defects which can decrease the product quality, for example in strip rolling (see Fig. 2). These flatness defects may have various origins, particularly thermal or/and mechanical. The present study focuses on numerical modeling of buckling phenomena occurring in strip rolling, the plate being submitted both to inter-stand tension and residual stresses. If the tension is sufficiently large, the strip may remain perfectly flat during the process and the flatness defects may appear only off-line, when the applied tension is released. Hence two instability problems will be considered in this paper: buckling under residual stress in the presence of a global tension and buckling due to the tension release in the presence of residual stresses.

Several models exist in the literature for buckling phenomena in thin strips due to residual stresses. This paper addresses particularly those involved in strip rolling. Since buckling relaxes residual stresses, Roddeman et al. [1–3] have presented a simple model based on a local relaxation of compressive residual stresses by introducing an additional in-plane deformation to the local strain field. This approach is widely used for membrane problems. A similar procedure has been used by Counhaye et al. [4] and implemented by Abdelkhalek et al. [5–8] in strip rolling models. This model leads to stress fields in agreement with experiments but it is intrinsically unable to predict geometrical flatness shapes.

Other papers focus on the detection of buckling loads due to increasing residual stresses. Most of them present analytical and/or semi-analytical studies in the simple case of a uniaxial residual stress field, the method being generally based on the Rayleigh–Ritz technique [9–13]. Nevertheless, Coman [14,15] was able to solve more intricate buckling problems via an analytical asymptotic method. General in-plane stress fields were accounted by Marchand [16] by using shell finite elements within the linear buckling subroutine of the code Abaqus. Some questionable hypotheses are often done in these works, for instance artificial boundary conditions to enforce wavy modes. In this paper we try to remove such artificial procedures.

To our knowledge, there are only few post-buckling analyses about strip instabilities due to residual stresses generated by a rolling process. Among them the work of Fischer et al. [12] mentioned previously. Another one is developed by Yukawa et al. [17] and relies on Riks method and plate finite elements. Also we find the model presented by Nakhoul et al. [18,19] which consists of a reduced-order technique that assumes a harmonic mode in the

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longitudinal direction and estimates the stress release due to buckling by combining bifurcation theory and one-dimensional finite elements. Buckling due to residual stresses is considered in these papers, but only Fischer et al. [12] studied buckling during tension release that is more difficult and more important in practice. This latter buckling problem will be treated in the present paper with a more advanced model.

This work describes a numerical model to compute post-buckling phenomena generated by a double loading combining residual stresses and global tension. It is designed for simulating the behavior of a strip during and after a rolling process. Especially the tension release leads to difficult post-buckling analyses that require an efficient path-following algorithm: the asymptotic numerical method (ANM), a powerful tool to solve nonlinear problems involving instabilities, has been chosen. Critical points and corresponding modes are computed using a bifurcation indicator. The effectiveness of such procedures have been established in previous works and for similar problems, for instance in [20–23]. The chosen shell finite element is also well established [24,25].

2. Basic numerical techniques

2.1. Shell model

The kinematic formulation is based on the classical plate and shell theory. The position vector **x** of a material point is expressed in the initial configuration as follows (see Fig. 3).

$$x(\theta_1,\theta_2,\theta_3) = r(\theta_1,\theta_2) + \theta_3 a_3(\theta_1,\theta_2) \tag{1}$$

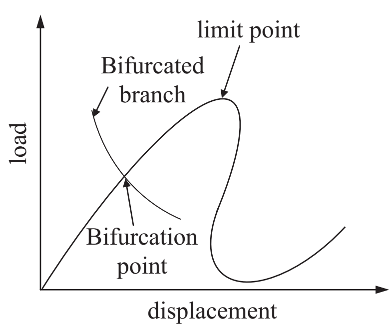

Fig. 1. A typical response curve in buckling analysis.

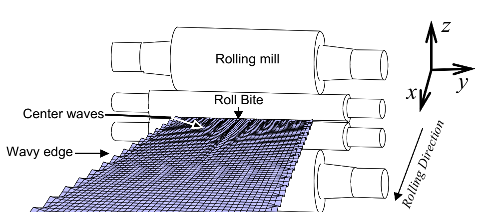

Fig. 2. Example of flatness defect observed in strip rolling.

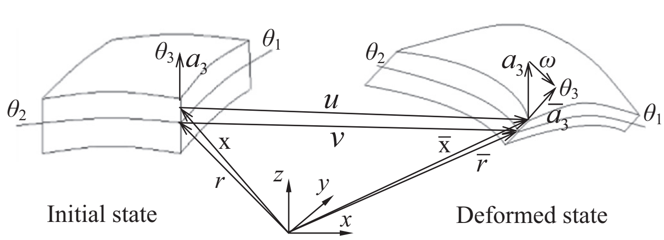

Fig. 3. Geometrical and kinematical description of the shell.

where r is the mid-surface vector and a_3 is the director vector of the surface in the considered point and $(\theta_1,\theta_2,\theta_3)$ represents the convective curvilinear coordinates. Assuming a linearly varying displacement in the thickness, this displacement is written as:

$$u(\theta_1,\theta_2,\theta_3) = v(\theta_1,\theta_2) + \theta_3\, \omega(\theta_1,\theta_2) \tag{2}$$

Variables v and ω represent respectively the mid-surface displacement and the difference between the director vectors in the deformed and the non-deformed configurations of the shell.

The Enhanced Assumed Strain (EAS) concept introduced in references [24–26] is used in this paper to improve the performance of elements. It is based on an enrichment of the deformation by introducing an additional strain field $\tilde{\gamma}$, independent of the displacement and chosen orthogonal to the stress field:

$$\begin{cases} \gamma = \gamma^c + \tilde{\gamma} \\ \gamma^c = \gamma^l(u) + \gamma^{nl}(u,u) \\ \int_\Omega S^t : \tilde{\gamma}\ d\Omega = 0 \end{cases} \tag{3}$$

where γ^c is the compatible Green-Lagrange strain decomposed into linear (γ^l) and non-linear (γ^{nl}) components. S is the second Piola-Kirchhoff stress tensor (t in upper index is transposition). $\tilde{\gamma}$ is chosen to ensure linear variation of the strain through the shell thickness [24].

2.2. A general strategy to solve the two-parameters buckling problem

The mechanical formulation is based on the Hu-Washisu functional which considers the displacement u, the strain γ and the stress field S as three independent variables. We consider a linear constitutive law in the present study.

Two kinds of mechanical loading (parameters) are considered in this model:

- Uniform edge load in the x-direction: in modeling flatness defects in thin strip rolling, the tension in the edge $\partial\Omega_3$ (see Table 1) is used to approach the rolling conditions (rolling tension).
- Residual stresses as internal loading for the structure: in strip rolling, residual stresses can be caused by heterogeneous plastic deformations or by heterogeneous thermal fields.

Hence, taking into account relations (3), the stationary condition of the Hu-Washisu functional leads to the following equations:

$$\begin{cases} \int_\Omega S^t : \delta\gamma^c d\Omega = \lambda^{(P)} \int_{\partial\Omega_3} P\delta u ds \\ \int_\Omega S^t : \delta\tilde{\gamma} d\Omega = 0 \\ S = \mathbb{C} : (\gamma^c + \tilde{\gamma}) + \lambda^{(S)} S^{res} \\ \gamma^c = \gamma^l(u) + \gamma^{nl}(u,u) \end{cases} \tag{4}$$

where $\lambda^{(P)}$ and $\lambda^{(s)}$ are two scalar parameters used to vary the loading levels corresponding respectively to the tension (P) and to the residual stress (S^{res}). $\mathbb{C}$ is the matrix of elastic constants.

The proposed model consists of four steps summarized in Table 1. Each step corresponds to a typical loading: a tension applied on the edge $\partial\Omega_3$ or a residual stress field. Generally, edge $\partial\Omega_1$ is clamped or pinned, edges $\partial\Omega_2$ and $\partial\Omega_4$ are simply supported, free or have symmetry planes and $\partial\Omega_3$ is simply supported.

Each step is solved using the ANM [20–22,25] and the finite element method as detailed in appendices. In step 2 the bifurcation indicator [20,21,23,25] is used to calculate the critical load $\lambda_c^{(S)}$ and the buckling mode. It is well adapted for problems where pre-buckling is nonlinear.