International Journal of Non-Linear Mechanics 71 (2015) 152–164

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Large deformations of nonlinear viscoelastic and multi-responsive beams

Anastasia Muliana

Department of Mechanical Engineering, Texas A&M University, United States

ARTICLE INFO

Article history:
Received 25 April 2014
Received in revised form
2 October 2014
Accepted 1 December 2014
Available online 10 December 2014

Keywords:
Nonlinear viscoelastic
Large deformations
Multi-responsive materials
Electro-active response
Polymers

ABSTRACT

This study presents analyses of deformations in nonlinear viscoelastic beams that experience large displacements and rotations due to mechanical, thermal, and electrical stimuli. The studied beams are relatively thin so that the effect of the transverse shear deformation is neglected, and the stretch along the transverse axis of the beams is also ignored. It is assumed that the plane that is perpendicular to the longitudinal axis of the undeformed beam remains plane during the deformations. The nonlinear kinematics of the finite strain beam theory presented by Reissner [27] is adopted, and a nonlinear viscoelastic constitutive relation based on a quasi-linear viscoelastic (QLV) model is considered for the beams. Deformation in beams due to mechanical, thermal, and electric field inputs are incorporated through the use of time integral functions, by separating the time-dependent function and nonlinear measures of field variables. The nonlinear measures are formulated by including higher order terms of the field variables, i.e. strain, temperature, and electric field. Responses of beams under mechanical, thermal, and electrical stimuli are illustrated and the effects of nonlinear constitutive relations on the overall deformations of the beams are highlighted.

1. Introduction

The development of flexible autonomous lightweight structures, which allow for controllable reconfigurations into various three-dimensional (3D) shapes, has many relevant applications in the fields of morphing air vehicles, deployable structures, flexible robots, etc. Typical flexible structures are macroscopically compliant and lightweight so that they can easily be deformed by mechanical and non-mechanical external stimuli, such as thermal, electrical, optical, and chemical stimuli. Polymers are appealing for many flexible structures since they are lightweight and compliant, compared to their metallic and ceramic counterparts, and can be easily fabricated and deformed into various shapes. The development of active polymers, such as shape memory polymers, piezoelectric polymers, dielectric elastomers, liquid crystal elastomers, etc., makes it possible to create flexible autonomous structures which can sense and adapt in response to various external stimuli. One of the main characteristics of polymers is their viscoelastic behavior; thus, it might be necessary to incorporate the viscoelastic response in analyzing performance of polymeric structures.

Many flexible structures are in the forms of thin filaments and sheets where flexure and stretching are dominating the deformations of these structures. These types of structures are typically modeled as rod, beam, plate, or shell. Flexible structures are often subjected to large displacements (large stretch and/or large rotation) during their services, whose motion cannot be sufficiently described based on linearized kinematics. Several studies have been conducted to describe motions in flexure-dominating structures undergoing moderate to large displacements. For example, von-Karman plate theory is an extension of Kirchhoff–Love plate theory that attempts on describing large displacements in linear elastic slender plates by incorporating the effect of lateral rotations on the in-plane axial (normal) deformations of the plate.[1] Reissner [27] formulated a finite strain beam theory undergoing plane deformations that accounts for finite bending and stretching. The nonlinear strain–displacement relations were consistent with the equilibrium equations for forces and moments. He also considered the transverse shear deformation effect in the finite strain beam formulation. For sufficiently small deformations and in absence of transverse shear effect, the Reissner finite strain beam reduces to

[1] The von-Karman theory leads to coupling in the axial-bending deformation, which results in nonlinear governing differential equations.

E-mail address: amuliana@tamu.edu

http://dx.doi.org/10.1016/j.ijnonlinmec.2014.12.001
0020-7462/

the classical Euler–Bernoulli beam. Epstein and Murray [7] presented governing equations and finite element solutions for elastic beams under large in-plane deformations. The nonlinear governing equation is defined based on a total Lagrangian formulation that satisfies objectivity, in which upon linearizing the displacements the superimposed rigid body motion should not lead to internal strains. They also discussed errors in determining the curvature when non-objective nonlinear formulation is considered. These errors increase with the angle of rotation. Hodges [13] discussed several curvatures that are consistent to nonlinear kinematics of large deformations of beams. These expressions of curvatures readily reduce to a linearized measure of curvature when the deformation and rotation are sufficiently small. Irschik and Gerstmayr [15] further presented a continuum mechanics based derivation of the Reissner finite beam theory for an originally straight beam made of a linear elastic material. They discussed two stress–strain measures, namely the Biot stress and strain, and the Piola-Kirchhoff stress and Green-St. Venant strain, with a linear elastic relation between the stress and its conjugate strain. It can be shown that the curvature and displacements presented in Irschik and Gerstmayr [15] are identical to the ones discussed by Epstein and Murray [7]. An elaborate discussion on flexible elastic bars with various boundary conditions can be found in Frisch-Fay [10].

Large displacements in flexural structures result in nonlinear governing differential equations, which may be challenging in obtaining exact analytical solutions. Several analytical and numerical methods have been presented to determine solutions of flexural structures of linear elastic materials undergoing in-plane large deformations, e.g., Epstein and Murray [7], Saje and Srpcic [29], Pak and Stauffer [24], Magnusson et al. [21], Banerjee et al. [3], Shvartsman [30], Vernerey and Pak [36], and Rungamornrat and Tangnovarad [28]. Saje and Srpcic [29] consider general polynomial functions for the axial and transverse displacements, which convert the nonlinear partial differential equations into a system of algebraic equations. These algebraic equations can be explicitly solved for obtaining the displacements. They also presented a finite difference method for solving the nonlinear differential equations. It is noted that nonlinear differential equations are often solved iteratively, whose converged solutions depend upon the initial guess (trial value) and multiple converged solutions are also possible. Shvartsman [30] used a direct method by converting the boundary value problems to initial value problems in order to obtain solutions to nonlinear differential equations of large deformation cantilever beams without the need of iterations.

Many flexible structures are made of viscoelastic polymers, whose response is time-dependent (or rate-dependent). Several studies have addressed deformations in viscoelastic beams, undergoing small displacement gradients. Both linear and nonlinear viscoelastic constitutive relations have been considered. Examples can be found in Wineman and Kolberg [39], Wang et al. [38], Rajagopal and Wineman [26], Paola et al. [6]. To our best knowledge, analyses of viscoelastic beams undergoing large in-plane displacements are currently limited. Some analytical and numerical studies of large deformation beams with *linear viscoelastic constitutive model* are discussed by Holden [14], Baranenko [4], Lee [20], Vaz and Caire [35]. Lee [20] and Vaz and Caire [35] used Runge–Kutta method for solving the integral-differential governing equations for linear viscoelastic cantilever beams undergoing large displacements. Beldica and Hilton [5] considered general nonlinear viscoelastic constitutive models for viscoelastic smart beams, and in order to incorporate large deformations, they considered the following measure of in-plane curvature $\partial^2 w/\partial x^2\left[1+(\partial w/\partial x)^2\right]^{-3/2}$ where w is the lateral displacement and x is the axis of the undeformed beam. As discussed by Hodges [13] and Epstein and Murray [7], and also presented in several literatures, the use of axis of the undeformed beam in the measure of the curvature does not account for the non-negligible axial displacement that corresponds to a large lateral displacement. Recently, Bahraini et al. [2] used fractional time derivative and finite element method for analyzing large deformations in viscoelastic beams. The one-dimensional viscoelastic constitutive model is expressed in terms of a differential operator, and the fractional derivative is used to explicitly express the time-dependent stress.

This study presents analyses of viscoelastic beams undergoing large in-plane displacements actuated by mechanical or non-mechanical stimuli (thermal and electric field inputs). The studies have some relevance to understanding folding/unfolding behaviors in flexible bodies, which find many applications in deployable structures, flexible robots, etc. We adopted the nonlinear kinematics in the finite strain beam theory of Reissner [27], whose continuum mechanics based derivation of a linear elastic beam, neglecting the transverse shear effect, has been presented by Irschik and Gerstmayr [15]. We consider a *nonlinear viscoelastic constitutive model* for the beam, which is based on a quasi-linear viscoelastic (QLV) model. The QLV model has been used to simulate nonlinear viscoelastic responses of biological tissues [8,22] and polymers [23,34]. We also consider nonlinear thermo-electro-active beams by incorporating higher order terms of the temperature and electric field inputs in the constitutive relations. The effect of energy dissipation from the viscous deformation and electric currents that is converted into heat, which could increase the temperature of the beam, is ignored. It will be discussed later in the manuscript that when sufficiently large external stimuli are prescribed, in addition to large deformations in the beam, the effect of nonlinear material responses of the deformation of the beam becomes significant. This manuscript starts with a brief discussion on the basic kinematics and equilibrium equations of an elastic beam undergoing large displacements in Section 2. We focus on a beam that is originally straight in its undeformed configuration. Section 3 presents the large displacements formulations of a nonlinear viscoelastic beam subjected to mechanical stimuli and Section 4 extends the formulations when the non-mechanical stimuli, such as thermal and electric inputs, are considered. Section 5 is dedicated to conclusions.

2. Large displacements of an elastic beam

Consider a straight prismatic homogeneous beam, in its undeformed configuration, whose longitudinal axis[2] is placed along the x-axis of the Cartesian coordinate system (Fig. 1). The x–y plane is the plane of symmetry with regards to the applied load and deformation, and the z-axis is the out of plane axis. We follow the kinematics of the deformed beam without any restriction to the magnitude of the displacements or strains, previously discussed in Reissner [27], Epstein and Murray [7], and Irschik and Gerstmayr [15]. It is assumed that the plane that is perpendicular to the longitudinal axis of the undeformed beam remains plane during the deformation. We neglect the effect of a transverse shear strain on the lateral deformation of the beam and ignore the stretch along the transverse axes of the beam. Consider a differential beam element, whose initial length is dx, and the current length along its centroidal axis is ds (see Fig. 1). The axial stretch ratio along the centroidal axis is

$$\Lambda_{xo}=\frac{ds}{dx}=\varepsilon_{xo}+1;\quad \varepsilon_{xo}=\frac{ds-dx}{dx} \tag{2.1}$$

where ε_{xo} is the engineering strain of the differential beam element along its centroidal axis. We now consider a material point that is originally located at a distance x along the centroidal

[2] This is a centroidal axis of the beam.