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A non-iterative transformation method for Blasius equation with moving wall or surface gasification

Riccardo Fazio

Department of Mathematics and Computer Science, University of Messina, Viale F. Stagno D'Alcontres, 31, 98166 Messina, Italy

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ABSTRACT

We define a non-iterative transformation method for Blasius equation with moving wall or surface gasification. The defined method allows us to deal with classes of problems in boundary layer theory that, depending on a parameter, admit multiple or no solutions. This approach is particularly convenient when the main interest is on the behaviour of the considered models with respect to the involved parameter. The obtained numerical results are found to be in good agreement with those available in the literature.

1. Introduction

The problem of determining the steady two-dimensional motion of a fluid past a flat plate placed edge-ways to the stream was formulated in general terms, according to the boundary layer theory, by Prandtl [1], and was investigated in detail by Blasius [2]. The engineering interest was to calculate the shear at the plate (skin friction), which leads to the determination of the viscous drag on the plate, see for instance Schlichting [3].

The celebrated Blasius problem is given by

$$\frac{d^3f}{d\eta^3}+Pf\frac{d^2f}{d\eta^2}=0$$

$$f(0)=\frac{df}{d\eta}(0)=0, \quad \frac{df}{d\eta}(\eta)\rightarrow 1 \text{ as } \eta\rightarrow\infty, \tag{1}$$

where f and η are suitable similarity variables and in the literature we can find either $P=1/2$ or $P=1$. This is a boundary value problem (BVP) defined on the semi-infinite interval $[0,\infty)$. According to Weyl [4], the unique solution of (1) has a positive second order derivative, which is monotone decreasing on $[0,\infty)$ and approaches to zero as η goes to infinity. The governing differential equation and the two boundary conditions at $\eta=0$ in (1) are invariant with respect to the scaling group of transformations

$$\eta^*=\lambda^{-\alpha}, \quad f^*=\lambda^{\alpha}f \tag{2}$$

where α is a non-zero constant: Töpfer used $\alpha=1/3$, see [5], but we have always put $\alpha=1$ in order to simplify the analysis. The mentioned invariance property has both analytical and numerical interest. From a numerical viewpoint a non-iterative transformation method (ITM) reducing the solution of (1) to the solution of a related initial value problem (IVP) was defined by Töpfer [5]. Owing to that transformation, a simple existence and uniqueness Theorem was given by Serrin [6] as reported by Meyer [7, pp. 104–105] or Hastings and McLeod [8, pp. 151–153]. Let us note here that the mentioned invariance property is essential to the error analysis of the truncated boundary solution due to Rubel [9], see Fazio [10].

Our main interest here is to extend Töpfer's method to classes of problems in boundary layer theory involving a physical parameter. This kind of extension was considered first by Na [11], see also Na [12, Chapters 8 and 9]. The application of a non-ITM to the Blasius equation with slip boundary condition, arising within the study of gas and liquid flows at the micro-scale regime [13,14], was considered already in [15]. Here we define a non-ITM for Blasius equation with moving wall considered by Ishak et al. [16] or surface gasification studied by Emmons [17] and recently by Lu and Law [18]. In particular, we find a way to solve non-iteratively the Sakiadis problem [19,20]. For the solution of the Sakiadis problem by an ITM see Fazio [21]. The defined method allows us to deal with classes of problems in boundary layer theory that, depending on a parameter, admit multiple or no solutions. This

E-mail address: rfazio@unime.it
URL: http://mat521.unime.it/fazio

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approach is particularly convenient when the main interest is on the behaviour of the considered models with respect to the involved parameter.

2. Moving wall

According to Ishak et al. [16] the differential problem governing a moving wall, with suitable boundary conditions, is given by

$$\frac{d^3f}{d\eta^3}+\frac{1}{2}f\frac{d^2f}{d\eta^2}=0$$
$$f(0)=0,\quad \frac{df}{d\eta}(0)=P,\quad \frac{df}{d\eta}(\eta)\to 1-P \quad \text{as } \eta\to\infty, \tag{3}$$

where P is a non-dimensional parameter given by the ratio of the wall to the flow velocities. Blasius problem (1) is recovered from (3) by setting $P=0$.

2.1. The non-ITM

The applicability of a non-ITM to the Blasius problem (1) is a consequence of both: the invariance of the governing differential equation and the two boundary conditions at $\eta=0$, and the non-invariance of the asymptotic boundary condition under the scaling transformation (2). In order to apply a non-ITM to the BVP (3) we consider P as a parameter involved in the scaling invariance, i.e., we define the extended scaling group

$$f^*=\lambda f,\quad \eta^*=\lambda^{-1}\eta,\quad P^*=\lambda^2 P. \tag{4}$$

Let us notice that, due to the given second boundary condition at $\eta=0$ and the asymptotic boundary condition in (3), P has to be transformed under the scaling group (4) with the same law of $(df/d\eta)(\eta)$. By setting a value of P^*, we can integrate the Blasius equation in (3) written in the star variables on $[0,\eta^*_\infty]$, where η^*_∞ is a suitable truncated boundary, with initial conditions

$$f^*(0)=0,\quad \frac{df^*}{d\eta^*}(0)=P^*,\quad \frac{d^2f^*}{d\eta^{*2}}(0)=\pm 1, \tag{5}$$

in order to compute an approximation $(df^*/d\eta^*)(\eta^*_\infty)$ for $(df^*/d\eta^*)(\infty)$ and the corresponding value of λ according to the equation:

$$\lambda=\left[\frac{df^*}{d\eta^*}(\eta^*_\infty)+P^*\right]^{1/2}. \tag{6}$$

Once the value of λ has been computed, by Eq. (6), we can find the missed initial conditions

$$\frac{df}{d\eta}(0)=\lambda^{-2}P^*,\quad \frac{d^2f}{d\eta^2}(0)=\lambda^{-3}\frac{d^2f^*}{d\eta^{*2}}(0). \tag{7}$$

Moreover, the numerical solution of the original BVP (3) can be computed by rescaling the solution of the IVP. In this way we get the solution of a given BVP by solving a related IVP.

We remark here that the plus (for $P<0.5$) or minus (when $P>0.5$) sign must be used for the second derivative in (5). Moreover, the computation of a value at infinity is unsuitable from a numerical viewpoint and therefore we use a truncated boundary η^*_∞ instead of infinity. For the application of the method defined above, depending on the behaviour of the numerical solution, we used $\eta^*_\infty=10$ or $\eta^*_\infty=15$.

In Table 1 we list sample numerical results obtained by the non-ITM for several values of P^*. Here the D notation indicates that these results were computed in double precision. As mentioned before, the case $P^*=P=0$ is the Blasius problem (1). In this case our non-ITM becomes the original method defined by Töpfer [5]. For the Blasius problem, the obtained skin friction coefficient is in good agreement with the values available in the literature, see for instance the value 0.332057336215 computed by Fazio [22] or the value 0.33205733621519630, believed to be correct to all the sixteen decimal places, reported by Boyd [23]. The values shown in the last line of Table 1 are related to the Sakiadis problem [19,20] and were found by a few trial and miss attempts. For this problem, the obtained skin friction coefficient is in good agreement with the values reported by other authors, e.g., −0.44375 Sakiadis [19], −0.4438 by Ishak et al. [16], −0.44374733 by Cortell [24] or −0.443806 by Fazio [21].

Fig. 1 shows the solution of the Sakiadis problem, describing the behaviour of a boundary layer flow due to a moving flat surface immersed in an otherwise quiescent fluid, corresponding to $P=1$. Actually, this is a case of practical interest if we are considering the plate as an idealisation of an airplane wing. Let us notice here that by rescaling we get $\eta^*_\infty<\eta_\infty$.

Table 1
Moving wall boundary condition: non-ITM numerical results.

$\frac{d^2f^*}{d\eta^{*2}}(0)$	P^*	$\frac{df^*}{d\eta^*}(\infty)$	$\frac{d^2f}{d\eta^2}(0)$	P
1	−500	1.55D04	5.46D−07	−0.033393
	−100	2.34D03	9.42D−06	−0.044591
	−5	36.325698	0.005704	−0.159613
	−1.5	4.368544	0.205830	−0.522913
	−1.25	3.529165	0.290627	−0.548447
	−1	2.917762	0.376537	−0.521441
	−0.75	2.503099	0.430814	−0.427814
	−0.5	2.250439	0.431797	−0.285643
	0	2.085393	0.332061	0
	1	2.440648	0.156689	0.290643
	5	5.771518	0.028287	0.464187
	100	1.00D02	3.53D−04	0.499557
	500	5.00D02	3.16D−05	0.499960
−1	100	99.822681	−3.54D−04	0.500444
	10	9.433763	−0.011673	0.514568
	5	4.182424	−0.035939	0.544519
	2	0.528464	−0.248722	0.790994
	1.719	−4.73D−05	−0.443715	1.000027

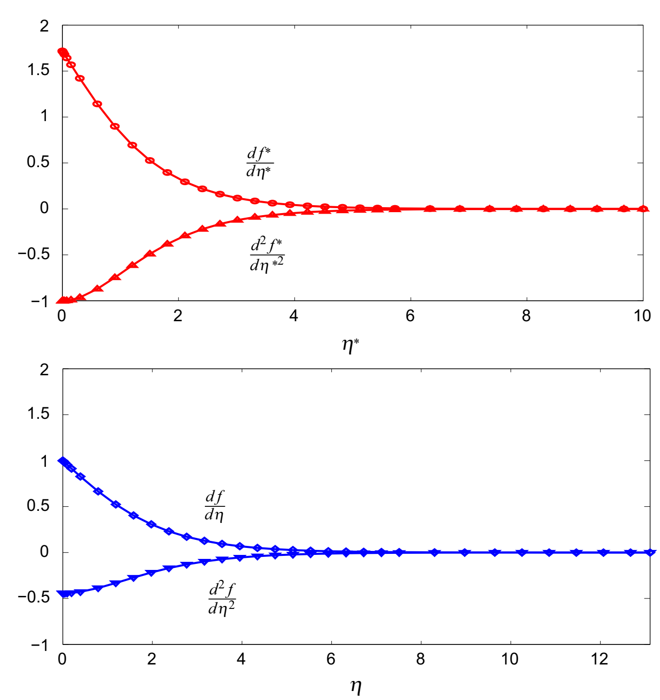

Fig. 1. Numerical results of the non-ITM. Top frame: solution of the IVP; bottom frame: solution of the Sakiadis problem found after rescaling.