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The effects of nonlinearities on the vibration of viscoelastic sandwich plates

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ABSTRACT

The nonlinear free and forced bending vibration of sandwich plates with incompressible viscoelastic core is investigated under the effects of different source of nonlinearities. For the core constrained between stiffer layers, the transverse shear strains, as well as the rotations are assumed to be moderate. The linear and quadratic displacement fields are also adopted for the in-plane and out-of-plane displacements of the core, respectively. The assumption of moderate transverse strains requires a nonlinear constitutive equation which is obtained from a single-integral nonlinear viscoelastic model using the assumed order of magnitudes for linear strains and rotations. The 5th-order method of multiple scales is directly applied to solve the equations of motion. The different-order linear partial differential equations that were obtained during the perturbation solution, are solved by the method of eigenfunction expansion and the nonhomogeneous boundary conditions are dealt with by transforming to homogeneous boundaries, or using the extended Green's formula. The effects of different system parameters on the nonlinear estimation of frequencies, damping ratios, and peak response are studied. Also, the importance of different nonlinear terms arisen from different ordering assumptions is assessed and the ranges of system parameters with higher values of error are identified.

1. Introduction

Needs for lighter structures with higher stiffness and desirable mechanical properties, have made sandwich structures an attractive and suitable type of construction for various engineering applications such as naval, aerospace and civil industries. The performance of such structures under dynamic loadings could be improved by using high damping viscoelastic materials between stiffer layers.

Studies on the dynamic behaviors of three-layer sandwich structure with a viscoelastic core, originated with the work of Kerwin [1] on the transverse traveling wave of the 3-layer sandwich beam. His study was based on the linearized equations of motion, which has been followed thus far by many other researchers to address different issues concerning the behavior of sandwich structures that arise in different situations. The linearization, however, would not be accurate enough if the displacements do not remain small. In this case, the contribution of nonlinear terms in kinematic relations (geometric nonlinearities), or constitutive relations (physical nonlinearity) becomes considerable. Extensive research studies have been performed in this respect to account for the effects of nonlinearities specifically in kinematic formulations of sandwich plates and shell. Some of these studies [2–6] developed the geometrically exact formulation which could be used to accurately model the highly nonlinear large deformations in applications such as very flexible composite robot arms or satellite appendages [5]. On the other hand, there are another group of studies that have included the geometrical nonlinearities based on Green's strain formula usually accompanied with the assumption of moderate or large transverse rotations and small stains in the plates and shells [7–10]. The theories based on these assumptions are specifically useful for moderate bending deformation of plates and shells and may not be accurate enough to be used in large deformations mentioned above. The basis of these theories is on the physical evidence that in the shell- and plate-type structures under bending deformation, it is usually the linear, transverse rotation terms that increase in bending deformation and give rise to the geometric nonlinearity. This assumption is often accompanied by small strains. However, in a soft layer, which is constrained between stiffer ones, it can be expected that the transverse shear strains will raise to moderate values, while the deformation amplitudes are in the same range. In this situation, the physical nonlinearities will become important.

For sandwich structures with viscoelastic layers, the exact geometrical formulation has not been widely used in the literature. The recent works by Lacarbonara and Pasquali [3] and Deu et al. [11], could be mentioned in this regard. Research studies based on moderate rotations are however older and more mature.

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In the earlier studies performed by Kovac et al. [12] and Hyer et al. [13,14], the nonlinear frequency–response of a damped three layer beam under harmonic transverse excitation was theoretically predicted and compared by experimental results. More refined theory compared to Ref. [12] was used in Ref. [13], by the First order Shear Deformation Theory (FSDT) for the core and the Classical Plate Theory (CPT) for the face layers, while the transverse normal deformations were neglected. Similar modeling approach with the Kelvin–Voigt viscoelastic model was adopted by Xia and Lukaziewicz [15,16] for studying the free and harmonically forced nonlinear vibration of sandwich plates. In their study on the forced vibration of a plate with the simply-supported moveable edge [15], only one linear mode is used with Galerkin's method to solve all the in-plane and transverse equations of motion. The nonlinear free vibration analysis of sandwich beams with the Finite Element Method (FEM) was performed by Ganapathi et al. [17]. They introduced an amplitude-dependent equivalent structural loss factor to characterize the dissipation capability of the structure in the presence of the geometric nonlinearity. Attempts to achieve an efficient and accurate solution for the nonlinear vibration of sandwich beams and plates without any restriction on the viscoelastic model have been made recently by Bilasse et al. [18,19]. Neglecting the in-plane inertia terms, they solved the in-plane equations by FEM and obtained the in-plane displacements in terms of transverse displacement. For solving the out-of-plane equations, the one-term Galerkin's method coupled with the harmonic balance method is used. In another study, Jacques et al. [20] proposed a more accurate solution for the nonlinear vibration of sandwich beams by using the harmonic balance and FEM. They showed that the nonlinear mode shape is exactly the same as the linear mode shape for a symmetric sandwich beam with simply-supported immovable edge. The case of unsymmetric configuration was not, however, considered. In all the aforementioned studies, the moderate rotation assumption was accounted for, only by the Von Karman nonlinearities, and the other nonlinear terms that would contribute to the kinematic relations were excluded.

The physically nonlinear viscoelastic model has also been used in the studies of Lee [21] and Gandhi [22] on the free and forced vibration of sandwich beams. In both studies, however, the nonlinear viscoelastic behaviors were described by empirical approaches and the role of transverse shear strains in raising nonlinear terms were not considered.

In the present study, a more complete moderate rotation nonlinear sandwich plate theory is incorporated by systematically deriving the approximated nonlinear kinematic and constitutive relations based on the assumption that both transverse rotations and shear strains are moderate in the core. For this purpose, following the procedure used in Refs. [7,9,23], the simplified kinematic equations are obtained from the Green's strain–displacement relations by assuming different order of magnitudes for different components of linear strain and rotation tensors. A similar procedure had been implemented by Casey and Naghdi [24] to develop a physically nonlinear approximated theory of elasticity. This procedure is also used in the present study to obtain a simplified constitutive equation from the three-dimensional form of the nonlinear, viscoelastic model proposed by Stafford [25], and Nambudiripad and Neis [26]. This model is a single-integral model which is obtained from the Green–Rivlin model based on a modified superposition principle (MSP) [26]. It must be emphasized here that the nonlinear model obtained in this way, could only be used in moderate deformations. For large deformation analyses more accurate theories based on the exact geometrical modeling must be used at the expense of more complex equations of motion.

The required formulation for mathematical modeling of the core is completed by defining the through-the-thickness variation of the core's displacement, which is assumed to be linear for the in-plane components, and quadratic for the out-of-plane components. The quadratic out-of-plane displacement field is found to be necessary for accurate modeling of the core, which is treated as an incompressible three-dimensional medium. For modeling the face layer, however, the CPT is employed, with the Von Karman nonlinearity. The constitutive modeling is also performed by using the plane-stress Hook's law. The governing equations consist of seven nonlinear Integro-Partial Differential Equations (IPDEs), which are obtained by the Hamilton's principle, and two equations corresponding to the incompressibility conditions. The free damped vibration and also the primary-resonance solutions are provided by direct application of the Multiple Scales Method (MMS). In direct application of MMS, 9 linear PDEs with some non-homogeneous BCs will appear in higher order approximations. To solve these equations by the method of eigenfunction expansion (MEE), some of nonhomogeneous BCs are homogenized by using changes of variables. The remaining nonhomogeneous BCs, are also treated by Green's formula [27] which is extended in the present study for a system of equations with more than one dependent and independent variables. The solution is obtained up to 5th order with the help of the method of reconstitution. Exploiting the explicit relations obtained in this way, some parametric studies are conducted and the differences that may appear with using different nonlinear terms in kinematic and constitutive relations are investigated for different ranges of system parameters. Moreover, the accuracy of using only one linear mode for the main transverse displacement is assessed for unsymmetrical configurations.

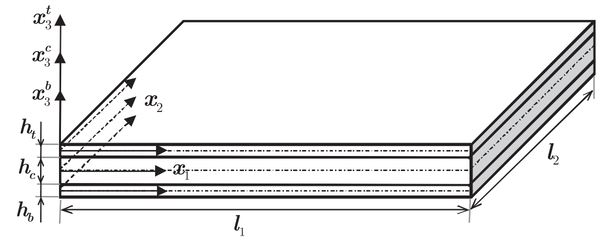

Fig. 1. The geometry of a sandwich plate and the coordinate systems adopted.

2. Formulation

The three layer rectangular sandwich plate considered in this study is shown in Fig. 1. As can be seen from this figure, different coordinate systems are adopted for each layer with their x_1x_2-plane coinciding with the mid-plane of that layer and the origins located at the corner of the plate. Moreover, the quantities with the superscripts or subscripts, t, c and b are related to the top layer, core and the bottom layer, respectively.

2.1. Kinematics

According to the method employed in Refs. [23,24], in order to achieve a simplified strain–displacement relation for the core, the Green strain tensor is written in terms of the linear strain tensor, $\mathbf{e}$, and the linear rotation tensor, $\mathbf{\Omega}$, as,

$$\mathbf{E}=\mathbf{e}+\frac{1}{2}(\mathbf{e}^2+\mathbf{e}\mathbf{\Omega}-\mathbf{\Omega}\mathbf{e}-\mathbf{\Omega}^2), \tag{1}$$

where $\mathbf{e}$ and $\mathbf{\Omega}$ are related to the displacement gradient matrix $(\partial\mathbf{U}/\partial\mathbf{X})$ of the core as

$$\mathbf{e}=\frac{1}{2}\left[\frac{\partial\mathbf{U}}{\partial\mathbf{X}}+\left(\frac{\partial\mathbf{U}}{\partial\mathbf{X}}\right)^T\right],\quad \mathbf{\Omega}=\frac{1}{2}\left[\frac{\partial\mathbf{U}}{\partial\mathbf{X}}-\left(\frac{\partial\mathbf{U}}{\partial\mathbf{X}}\right)^T\right], \tag{2}$$