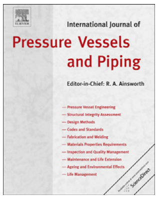
International Journal of Pressure Vessels and Piping 137 (2016) 22–27

Contents lists available at ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp



A novel approach to forced vibration behavior of thick-walled cylinders

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ARTICLE INFO

Article history:
Received 23 September 2014
Received in revised form
15 April 2015
Accepted 20 April 2015
Available online 29 April 2015

Keywords:
Anisotropic cylinder
Vibration
Pressure vessel members
Laplace transform
Durbin's numerical inversion

ABSTRACT

This study is to investigate the effect of anisotropy on the forced vibration behavior of hollow cylinders under dynamic internal pressure. The problems are solved analytically in the Laplace domain, and the results obtained are transformed to the real-time space using the modified Durbin's numerical inversion method. Durbin's numerical inverse method into the analysis of transient thermal stresses in annular fins is a novel approach. Durbin's numerical inverse method successfully implements the boundary value problem which can be solved in Laplace space. Various material models from the literature are used and corresponding radial displacement distributions and stress distributions are computed. Verification of the proposed method is done using benchmark solutions available in the literature for some special cases and virtually exact results are obtained. The anisotropy constant is a useful parameter from a design point of view in that it can be tailored for specific applications to control the stress distribution.

1. Introduction

Pressure vessel structural members such as cylinders, disks and spheres find broad application fields in the industry and their vibration analyses are deemed necessary for safe design and operation. Hollow cylinder and thick-walled cylindrical shell are common components in structural applications and device systems involving aerospace and submarine structures, civil engineering structures, machines, pipes, sensors and actuators, etc. Therefore, radial displacement and hoop stresses have also become significant in design problems related to pressure vessel through their effects on material properties and system parameters for decades. Many transform methods often require complicated techniques and elaborate work to overcome mathematic difficulties for radial displacement and hoop stress problems. The Durbin's numerical inversion technique which is a numerical method based on a Fourier series expansion provides an alternative approach to such problems. Since it was developed by Durbin [1], the method has been applied to various fields. For example, Ghadi et al. [2]. introduced the method for analytical solution for two-dimensional coupled thermoelastodynamics in a cylinder. There have been many studies, such as Refs. [3–8], focused thermal stresses in isotropic homogeneous rectangular plates. Exact vibration analyses of homogeneous cylinders using Laplace transform date back as early as 1940's [9]. Among related works those of Mirsky [10], Klosner and Dym [11], Ahmed [12] may also be cited and additional references can be found. A more detailed study on the subject is due to Ghosh [13] where axisymmetric vibration of thick cylinders under various continuous dynamic pressures is investigated.

An exact elasticity approach has been attempted by Bickford and Warren [14] where the solution is obtained due to a step-type pressure in the Laplace domain and the inverse transformation into the time domain is performed approximately using asymptotic expansions of Bessel's functions. Huang [15] has obtained exact expressions using Laplace transform for free and forced vibrations behavior of anisotropic cylinders subjected to harmonic pressure but no numerical results or parametric studies have been presented. Axisymmetric free vibrations of anisotropic cylinders of finite length has been studied in Ref. [16] using the Ritz method with power series approximations. Ding et al. [17]. has considered the dynamic response of an inhomogeneous, transversely isotropic, hollow sphere subjected to instantaneous constant internal radial pressure and thermal load by using the method of separation of variables and orthogonal functions. Analytical solutions to benchmark problems provide invaluable checks on the accuracy of numerical or approximate analytical schemes and allow for widely applicable parametric studies [18]. Stress

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http://dx.doi.org/10.1016/j.ijpvp.2015.04.015
0308-0161/

concentration factors (SCFs) for isotropic and orthotropic plates with a circular cutout under uniaxial and biaxial tension have determined by Wu and Mu [19]; they showed that the SCFs depend only on the cutout diameter to plate width ratio. Kokan and Gramoll [20] have analyzed thermal and mechanical stresses of cylindrical shells due to the tension force during winding procedure. In addition, several analyses have been carried out concerning three-dimensional free vibrations of thick circular and sandwich cylindrical shells. Malekzadeh et al. [21]. have analyzed the free vibration of a three-dimensional cylindrical shell with various boundary conditions and on the Winkler/Pasternak elastic medium. Zhou et al. [22] have developed an approach based on Chebyshev polynomials and Ritz method for evaluating natural frequencies of solid and hollow cylinders. Sofiyev [23] has discussed the vibration and buckling of sandwich cylindrical shells covered by different types of coatings, such as functionally graded, metal and ceramic coatings and subjected to a uniform hydrostatic pressure.

Recently, an energy-oriented modified Fourier method can also be used to solve the tiled problem. Vibration of plates and shells with general boundary conditions has been carried out by this method. For example, Ye et al. [24]. is applied to the free vibration analysis of 3D thick cylindrical shells with general end conditions and resting on elastic foundations. Su et al. [25] have studied three-dimensional vibration analysis of thick FG conical, cylindrical shell and annular plate structures with arbitrary elastic restraints. Jin et al. [26] have presented the vibration of laminated cylindrical shells with general elastic boundary conditions by a modified Fourier method. The exact 3D solutions for the free vibration of FG rectangular plates with general boundary conditions has been also provided by Jin et al. [27] using Rayleigh–Ritz method. A unified solution method for the free vibrations of FGM cylindrical, conical shells and annular plates with general boundary conditions based on the first-order shear deformation theory (FSDT) has been presented by Su et al. [28].

A novel approach will be attempted to obtain radial displacement distributions and hoop stresses in a simple and efficient manner: the Durbin's numerical inverse method will be infused into the analysis to convert the problem to a general boundary condition problem which can efficiently be solved. In this article, an attempt is being made to find the solution of the axisymmetric forced vibrations for a thick-walled orthotropic cylinder under the plane-strain condition. We normalize the governing partial differential equation subject to appropriate initial, boundary and interior condition. The governing equations are decoupled using the Laplace transform with respect to time. Next the general solutions for displacements are obtained in the transform domain. Then we complete the inversion to the real domain using a Fourier technique. The dynamic responses of radial displacement and hoop stresses of the hollow cylinder in the real domain are presented numerically. Various internal dynamic pressures are considered and the theoretical foundation is provided for further analyses with virtually any pressure expressed in terms of elementary functions. For an internal dynamic pressure continuous with time analytical solutions may be obtained through the calculus of residues. A benchmark solution for such a pressure will be obtained and it will be used to verify the numerical procedure. The method chosen in the present work, Durbin's numerical inverse Laplace transform method, has been efficiently implemented in vibration analysis of structural elements (e.g. see Temel [29] and Temel et al. [30]). The results are presented for dynamic stresses. In doing this, a parametric study is performed for an anisotropy parameter which is a stiffness ratio defined as the square root of the ratio of circumferential stiffness to radial stiffness to indicate the degree of anisotropy.

2. Analysis

2.1. Governing equations

Consider a thick-walled hollow cylinder of inner radius a and outer radius ka where k is a constant (see Fig. 1.). The cylinder is composed of a material exhibiting polar orthotropy. Under axisymmetric conditions the strain-displacement relations in terms of the radial displacement u are

$$\varepsilon_r = \frac{\partial u}{\partial r} \quad \text{and} \quad \varepsilon_\theta = \frac{u}{r} \tag{1}$$

and the stress–strain relations are

$$\begin{aligned} \sigma_r &= C_{11}\varepsilon_r + C_{12}\varepsilon_\theta \\ \sigma_\theta &= C_{12}\varepsilon_r + C_{22}\varepsilon_\theta \end{aligned} \tag{2}$$

where C_{11} and C_{22} refer to stiffness in the radial and circumferential directions, respectively and C_{12} includes the Poisson's effect. The only nontrivial equilibrium equation is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = \rho \frac{\partial^2 u}{\partial t^2} \tag{3}$$

Using Eqs. (1–3) the governing equation of radial displacement becomes

$$\frac{r^2 \partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} - n^2 u = \frac{r^2}{c^2}\frac{\partial^2 u}{\partial t^2} \tag{4}$$

where $c^2 = \frac{C_{11}}{\rho}$, $n^2 = \frac{C_{22}}{C_{11}}$, ρ is the material density.

With inner pressure $P(t)$, the boundary conditions are given as:

$$\sigma_r|_{r=a} = -P(t) \quad \sigma_r|_{r=ka} = 0 \tag{5}$$

Using the dimensionless variables

$$v = \frac{u}{a}, \quad x = \frac{r}{a}, \quad \tau = \frac{ct}{a} \tag{6}$$

Renders Eq. (4) in the form

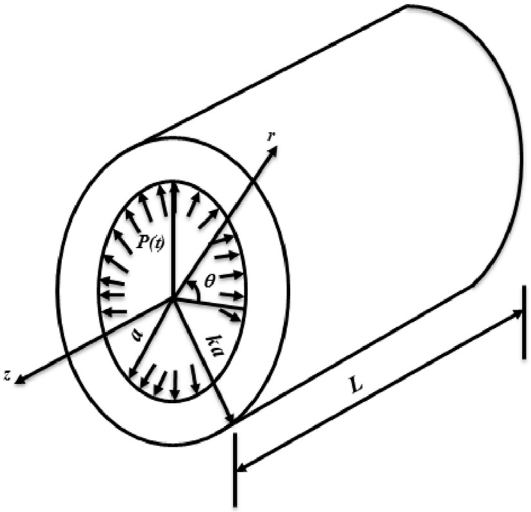

Fig. 1. Schematic diagram of a thick-walled cylinder.