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Mathematical modeling for quantitative estimation of geometric effects of nearby rebar in electrical resistivity measurement

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ABSTRACT

Concrete resistivity in reinforced concrete structures is used as an indicator of durability related to the corrosion risk. This study aims to develop a quantitative analysis method of geometric effects of nearby reinforcement in the electrical resistivity measurement. The A-REM proposed in this study is a mathematical model to enable analysis of not only concrete (or mortar) and reinforcement resistivity, but also of geometric effects such as rebar diameter, cover depth, electrode interval, and the spatial relationship between the rebar and electrodes. The effect of nearby reinforcement on the electrical resistivity measurement was presented to be obtained from the apparent resistivity rate (AR rate) calculated by the A-REM. The A-REM for the estimation of the nearby reinforcement effects was verified by comparing the apparent resistivity values obtained from mortar specimens having different reinforcement diameters and cover depths. The result shows that the calculated values of the A-REM are correlated with the experimental values of apparent resistivity.

1. Introduction

Electrochemical methods of diagnosing steel corrosion include the half-cell potential, polarization resistance, and A.C. impedance [1–3]. And the electrical resistivity method is used as an evaluation of steel corrosive environment by measuring concrete resistivity, which is an index of the moisture condition of concrete [4–6].

The resistivity method is advantageous since it measures concrete resistivity in a non-destructive examination without damaging the surface of the concrete. There have been various studies based on this method for the estimation of corrosion in concrete structure [7–11]. However, for estimation of concrete resistivity, reinforcement detection must be performed to ensure that resistivity measurements are taken at sites at which the lower sections are reinforcement-free.

A recent study has been conducted to develop techniques to estimate concrete resistivity in areas above reinforcement [12–15]. The proposed Resistivity Estimation Model (REM), based on the method of image charges, is a mathematical model that takes into account various factors such as concrete resistivity, steel resistivity, electrode interval, and geometric conditions including reinforcement diameter and concrete cover depth [16,17]. The proposed model, however, is only capable of estimating concrete resistivity in areas directly above reinforcement, and cannot eliminate geometric effects of nearby reinforcement. In order to increase the reliability of the resistivity measurement in the concrete structure, the influence of the surrounding reinforcement as well as the reinforcement directly under the measurement location must be considered.

This study proposes a new model to analyze geometric effects of nearby reinforcement in the other areas in addition to those directly above reinforcement. The proposed Advanced Resistivity Estimation Model (A-REM) expands the scope of analysis of the previous REM. Furthermore, this new method's validity is evaluated through a comparison of the theoretical results and experimental values.

2. Theoretical background

2.1. Principle

To estimate the resistivity of concrete, it is standard to employ Wenner electrode array [18], which consists of four collinear, two

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outer current, and two inner potential electrodes; these electrodes are arranged in a line at equal intervals, a, as shown in Fig. 1.

The formula for resistivity under the Wenner method is as follows:

$$\rho = 2\pi a \frac{V}{I} \tag{1}$$

where ρ is the resistivity, I is the current source, V is the potential difference, a is the electrode interval, and $2\pi a$ is the geometric factor that represents the geometric characteristics of the Wenner electrode array.

However, Eq. (1) is used to estimate the resistivity of homogeneous media, and cannot be applied to heterogeneous media like concrete structures. To deal with this problem, the concept of apparent resistivity, ρ_a , is introduced, as shown in Eq. (2) [19]; this concept allows resistivity analysis of heterogeneous media.

$$\rho_a = 2\pi a \frac{V_a}{I} \tag{2}$$

2.2. Resistivity estimation model (REM) above reinforcement

The apparent resistivity of a complex body comprised of media having different resistivities allows an analysis of the distribution states; however, it is difficult to estimate the exact resistivity of each medium.

The Resistivity Estimation Model (REM), a mathematical model based on the method of image charges, was proposed to consider the effects of reinforcement in concrete to analyze the apparent resistivity [16]. In employing the method of image charges, the resistivity reflection coefficient was defined as the relative position and the ratio of the density of the current source to that of the image charge. This coefficient represents the relationship between a current source at the boundary of media having different resistivity and the image current source on the opposite side. As shown in Fig. 2 [16], this model uses the Wenner electrode array, which consists of two outer current electrodes and two inner potential electrodes spaced at a distance of a. In Eq. (3), the REM is determined by various factors including concrete resistivity, reinforcement resistivity, cover depth, reinforcement diameter, and electrode interval. The REM enables an analysis of the concrete resistivity and the reinforcement resistivity by measuring directly above reinforcement.

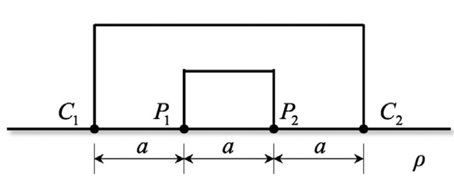

Fig. 1. Resistivity measurement method: Wenner Array.

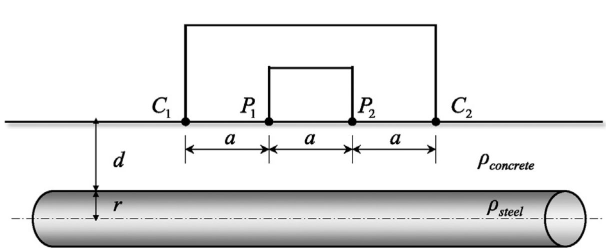

Fig. 2. Resistivity Estimation Model (REM) above rebar.

$$\rho_{REM} = \frac{V_{REM}\cdot\pi a}{I} = \rho_1\left[\frac{1}{2} + \sum_{n=1}^{\infty}\left[\begin{array}{l}\left[K_n\prod_{n=1}^{\infty}\frac{Q_n}{K_n}\right]\cdot\left[\frac{1}{\left(1+H_n^2\right)^{\frac{1}{2}}} - \frac{1}{\left(4+H_n^2\right)^{\frac{1}{2}}}\right] \\ + \left[\prod_{n=1}^{\infty}\frac{Q_n}{K_n}\right]\cdot\left[\frac{1}{\left(1+G_n^2\right)^{1/2}} - \frac{1}{\left(4+G_n^2\right)^{1/2}}\right]\end{array}\right]\right] \tag{3}$$

where

$$K_n = \frac{r}{(1+2(n-1))d + r} \tag{4}$$

$$Q_n = \frac{K_n(\rho_2 - \rho_1)}{\sqrt{K_n}\rho_2 - \rho_1} \tag{5}$$

$$H_n = \frac{d + r(1-K_n)}{a} \tag{6}$$

$$G_n = \frac{2nd}{a} \tag{7}$$

and

ρ_{REM} = apparent resistivity (Ω·m), obtained by REM,
V_{REM} = potential difference (V), obtained by REM,
I = current source (A),
$\rho_1 (= \rho_{concrete})$ = concrete resistivity (Ω·m),
ρ_2 = reinforcement resistivity (Ω·m),
d = cover depth (m),
r = reinforcement radius (m), and
a = intervals between electrodes (m).

3. Advanced resistivity estimation model (A-REM)

3.1. Resistivity reflection coefficient

A concrete structure, a complex body of concrete and reinforcement, consists of media with different resistivities and forms.

Based on the method of image charges, this study involves assumptions for the sizes and positions of a current source in the concrete surrounding reinforcement, and for the generated image charge in cylindrical reinforcement. To make these assumptions, it is necessary to determine the position of the image charge in relation to the Apollonian circle and the two-layer structure, and the magnitude of the image charge density based on the resistivity reflection coefficient.

To begin with, an Apollonian circle, which is used in modeling with the cylindrical model, is used to determine the relative position of the point current source I and the image charge I'.

As shown in Eq. (8), the proportionality coefficient, k, expresses the relationship between point I outside of the circle and point I' on line $\overline{OI}$ and within the circle as $l : r = r : c$, where l is the distance from the origin (O) to point I, r is the radius, and c is the distance from O to I' as shown in Fig. 3 [16].

Accordingly, the positions of the point current source I and the image charge I', generated on the circular cross-section with respect to the origin as a datum point, are relatively determined using proportionality coefficient k in Eq. (8) along the Apollonian