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Upscaling the elastic stiffness of foam concrete as a three-phase composite material

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ABSTRACT

The stiffness of foam concrete depends primarily on the added porosity. Nevertheless, by performing 3D elastic numerical simulations on artificial unit cells in the frame of periodic homogenization, it is shown that describing foam concrete as a porous material is not sufficient to explain the experimental measurements of the Young modulus for added porosity higher than 40%. Indeed, introducing sand as a third phase enables to recover accurate estimates of the Young modulus. Furthermore, for highly porous concrete foams, it is shown that the stress concentrates in thin members deprived of stiff sand particles, thus leading to a softer overall stiffness.

1. Introduction

Foam concrete, also named cellular concrete, offers many advantages as a building material. Indeed, it features a low density, a low thermal conductivity and a valuable fire and water resistance [1,2]. Lastly, it can easily be manipulated, pumped and cast into various shapes [3]. Regarding the formulation of foam concrete, the volume fraction of void, hereby called the porosity, can be tuned to trade cost, density and insulation properties for durability, strength and stiffness [1,3-7]. Typical uses include floors, trench fills, roof insulation and masonry units [8].

For lightweight concrete containing soft EPS beads formulated in [4], extending [9], it has been shown that the Young modulus does not depend on the size of the beads in the range 1 mm–6.3 mm, for porosities in the range 0%–50%. As a result, the Young modulus mainly depends on the porosity, exhibiting a one-to-one relationship in this domain. In addition, different mean-field homogenization schemes have been defined with the view to explain the dependence of the Young modulus to the added porosity, thus aiming at a scientific modelling of the stiffness. For instance, the fact that the experimental measurements of the Young modulus are compliant with the Hashin–Shtrikman upper bound [10] has been checked and the trisphere model [11,12] has been successfully applied to provide a better estimate of the stiffness [9]. Furthermore, the differential scheme [13] ensures a good agreement on a wider range of porosity ($0 \leq p \leq 56\%$) as the Young modulus varies directly with $(1-p)^2$ [14]. This exponent of 2 is consistent with expected and measured trends for foams where the bending is mainly attributed to the bending of the cell ribs [15,16]. Nevertheless, for foam concretes featuring high porosities (up to 75%), power laws featuring exponents higher than 2 ($E = 24$ GPa $(1-p)^{2.5}$ [17] or $E = 32.9$ GPa$(1-p)^{2.8}$ [6]) have been fitted to experimental results, thus questioning the use of the differential scheme for porous materials.

As an alternative to Eshelby-based mean-field schemes, full-field numerical elastic simulations can be performed on an artificial unit cell of periodic composite materials to estimate the effective overall stiffness [18-21]. Two steps are involved in the process. First, an artificial periodic unit cell complying with the observed microstructure of the composite material must be built. Parameters such as volume fractions, pore size distributions and minimum spacing may be accounted for in this step. Then, elastic computations help estimating the effective stiffness of the composite material by relying on the periodic homogenization theory.

The description of foam concrete as a biphasic material will be questioned by performing numerical simulations. First, concrete foam is considered as a biphasic material and the Young modulus will be estimated for different volume fractions, pore size distributions and parameters driving the geometry of the microstructure. These estimates will be compared to experimental measurements to show that foam concrete can hardly be described as a biphasic composite material at high porosity. Then, sand is introduced as the third phase and the estimated Young modulus is compared to the experimental measurements.

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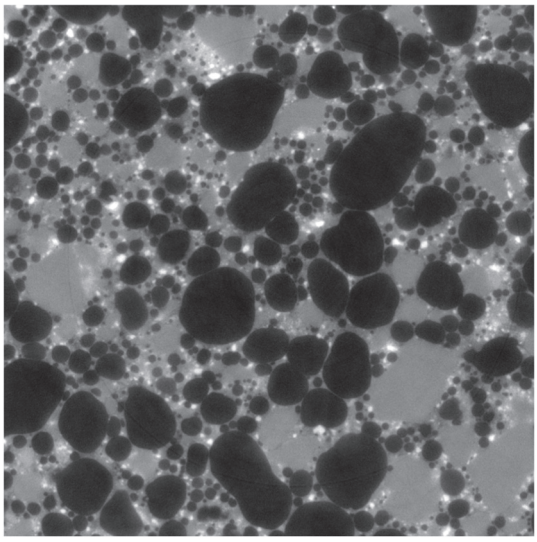

Fig. 1. A 6 × 6 mm^2 slice extracted from an X-ray tomographic image of a foam concrete of density 0.6 and porosity 70% [17], the pixel size being 6 µm. Dark areas are pores, grey uniform areas are sand particles, white dots are likely related to unhydrated cement particles.

2. 3D numerical modelling

2.1. Generation of artificial unit cells

So as to perform 3D numerical simulations, unit cells complying with the morphology of the considered material must be generated. By mean of X-ray tomography, foam concrete can visibly be described as porous matrix-inclusion material, where the pores are rather spherical (Fig. 1). In addition, the sieve curve of the sand and the pore size distribution are such that the sand particles are smaller than the pores (Fig. 2). Nevertheless, there is no separation of scale between the sand and the pores. In the present section, a procedure to obtain unit cells satisfying this description is proposed.

On the one hand, for matrix-inclusion composite materials, the random sequential adsorption algorithm [22] or the Lubachevsky-Stillinger algorithm [23] are often applied to pack spherical inclusions and reach the targeted volume fractions. On the other hand, the microstructure of closed-cell foams is often modelled by using Voronoi or Laguerre diagrams. In the present work, a continuum of microstructures between these geometries is defined to model closed-cell foams of any volume fractions. First, the Lubachevsky-Stillinger algorithm [23] is adopted to produce periodic unit cells featuring thousands of spherical inclusions and volume fractions up to about 60% [24]. This algorithm is briefly recalled in the next section.

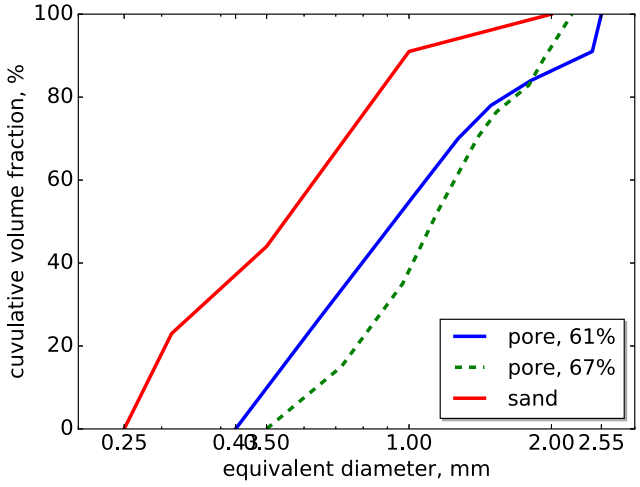

Fig. 2. Pore size distributions are extracted from tomographic images at the Navier Laboratory and the size distribution of sand grains is obtained by using sieves. The pore size distribution slightly depends on the amount of foam added to the mix, i. e. the porosity. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2.1.1. The Lubachevsky-Stillinger algorithm for periodic unit cells

The Lubachevsky-Stillinger algorithm [23,25] is an event-driven algorithm designed to pack hard spherical particles in a given volume. As input, it requires the size of the unit cell and both the targeted volume fractions ϕ_i and size distributions for each kind of particles $i \in$ (sand,pore).

The size distribution of sand particles or pores, named the sieve curve, is defined by the volume fraction of particles $P_i(v)$ of volume lower than the volume v (Fig. 2). For the shake of simplicity, it is modelled as a piecewise linear function on intervals $[v_j,v_{j+1}]$, $j \in 0...N_i^s$, the volume fraction of particles in these range being $p_{i,j}$. N_i^s is the number of sieves. A list of targeted volumes $v_{i,k}$, $k \in 0...N_i^u$ is randomly picked according to the sieve curve so as to fill a volume $\sum_k v_{i,k} = V\phi_i$, where V is the volume of the rectangular unit cell. N_i^u is the number of spheres of kind i to be placed in the unit cell. The Lubachevsky-Stillinger algorithm is seeded by randomly placing dots in the unit cell at time $t = 0$. Each dot becomes a sphere as time goes by, the growth of its radius being linear. Consequently, the growth rate is set to $a_{i,k} = (3/(4\pi)v_{i,k})^{1/3}$ so that all spheres reach their targeted volume $v_{i,k}$ at time $t = 1$.

The Lubachesky-Stillinger [23,25] enforces that the growing spheres do not overlap. Indeed, the spheres are allowed to move at uniform velocity and collisions are handled by changing these velocities at the time of collisions. The time of the potential collision can be analytically computed since the velocities are uniform and the growth rate is linear. Consequently, the algorithm can jump from one collision to the next without time-stepping. Other potential events are encounters with the boundaries of the unit cell, where instances of the sphere must be introduced to ensure periodicity [24,26]. Indeed, either the sphere hits a boundary and a new connected instance of the sphere must be created at the opposite boundary or an instance leaves the unit cell and it can be destroyed to save memory. An additional improvement consists in splitting the domain into rectangular sectors to reduce the number of potential events. Consequently, if N instances of different spheres overlap with a sector, $N(N-1)/2 + 6N$ potential events can occur in that sector.

The output of the Lubachevsky-Stillinger algorithm specifies the center, the volume and the kind of each sphere in the unit cell. The algorithm is very efficient at packing spherical particles as it can reach volume fractions of 74% for an unimodal pore size distribution. Nevertheless, the degree of order increases monotonically with the jammed packing fraction [27] and reaching high volume fractions ($\geq$64.5%) requires more time as the growth rates need to be reduced [25]. To avoid these shortcomings, an additional step allowing to reach any volume fractions of pores is introduced in the next section, though the pores become non-spherical.

2.1.2. A continuum of microstructures between spherical inclusions and Laguerre diagrams

Foams are often modelled by considering Laguerre diagrams, also called Power diagrams. These partitions of space into convex polyhedral cells are built starting from n seeds characterized by their position $\underline{x}_k$ and a scalar, named power, r_k. The cell i is defined as the set of points $\underline{x}$ such that

$$\|\underline{x} - \underline{x}_i\|^2 - r_i^2 \leq \|\underline{x} - \underline{x}_k\|^2 - r_k^2 \quad \forall\, k \tag{1}$$

If all powers r_k are equal, the resulting tessellation is a Voronoi