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Bending strength of porous ceramics tiles: Bounds and estimates of effective properties of an Intermingled Fractal Units' model

Michele Brun, Ludovica Casnedi, Giorgio Pia*

Dipartimento di Ingegneria Meccanica, Chimica e dei Materiali, Università degli Studi di Cagliari, Piazza d'Armi, 09123 Cagliari, Italy

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ABSTRACT

In this work, a semi-analytical model for the determination of effective mechanical properties of porous ceramic tiles obtained by pore forming agent is proposed. MIP tests allow measuring porosity and pore size distribution of experimental systems. These data are used for developing an Intermingled Fractal Units' model (IFU) as approximant of porous microstructures. IFU model is then combined with classical structural mechanics theory for the analytical computation of the bending strength of brittle ceramic beams. Bounds and estimates are given in full form and the detailed algorithm can be easily implemented in a numerical package. A preliminary comparison with experimental data shows the capability of the proposed model to reproduce the effective mechanical behaviour of ceramic tiles.

1. Introduction

Porous ceramics represent an important class of materials being applied in a large number of industrial and engineering fields [1]. Their unique properties make them particularly suitable for fabricating filters and membranes of particulates as well as hot corrosive gases [1,2], environmental friendly fuel cell electrodes [3,4], catalyst supports for biomaterials [5], piezoelectric materials [6] and acoustically as well as thermally insulating bulk media [7,8]. Indeed, in these applications, the required materials are characterised by relatively low mass [9,10], low fractional density [11], low thermal conductivity [12], resistance to chemical attack [13], high specific surface area [14,15], high permeability, resistance to high temperature and thermal cycling [16–18] and high mechanical properties [19,20].

In order to obtain the best performance of porous ceramics, particular attention has been paid to several fabrication methods such as gel casting process [21,22], the organic foam technique [23], the freeze casting method [24,25] and pore-forming agent method [12,26]. The last method is one of the most frequently used methods to obtain porous ceramic tiles [12,27,28]. It consists of mixing raw materials with additives, which burn out during sintering process leaving voids into the ceramic matrix. Manufactures can be obtained by compaction, applying a uniaxial or hydrostatic stress capable of increasing powder bed density and extrusion, moulding the paste using a specific matrix. Fabrication methods, sintering process (temperature, velocity etc.) and pore forming agent typology give rise to a porous structure influencing pore microstructural parameters such as size, size distribution, volume fraction and higher-order topological properties including orientation, three-dimensional shape, tortuosity etc. [29]. In particular, pore forming agent allows to obtain the formation of a fragmentary and irregular microstructure [30,31]. A large number of pore forming agents can be used and starch is often implemented. However, its application is limited to production of manufactories with large pores in the range of 5–50 μm [12,28,32,33]. In this sense, polymers represent a valid alternative. Principally, they can guarantee excellent workability of the mixture and result into low polluting and very cheap product [12]. Moreover, they burn out rapidly at a low temperature.

The possibility to control porous microstructure represents a crucial objective for improving physical properties such as heat conductivity, mass transfer, elastic and plastic behaviour. A particular attention must be paid to mechanical properties. The design of porous materials needs to balance all these aspects in order to obtain performing materials for specific applications. The presence of voids obstructs heat transfer and increases fluid permeability. At the same time, the mechanical behaviour is generally softened. On the contrary, a more dense material is more conductive, less permeable by fluids and stiffer to mechanical loads. In this sense, it is easy to note that mechanical properties are related to resistant surface or pore voids fraction [34–36]. However, mechanical resistance is also influenced by intrinsic characteristics of the solid phases [37–39].

For such two-phase composites, the separation of length scale holds such that 'microscopic' length scale is much larger than molecular dimensions, justifying the adoption of a continuum solid approach, but much smaller than the characteristic length of the macroscopic sample,

* Corresponding author.
E-mail address: giorgio.pia@dimcm.unica.it (G. Pia).

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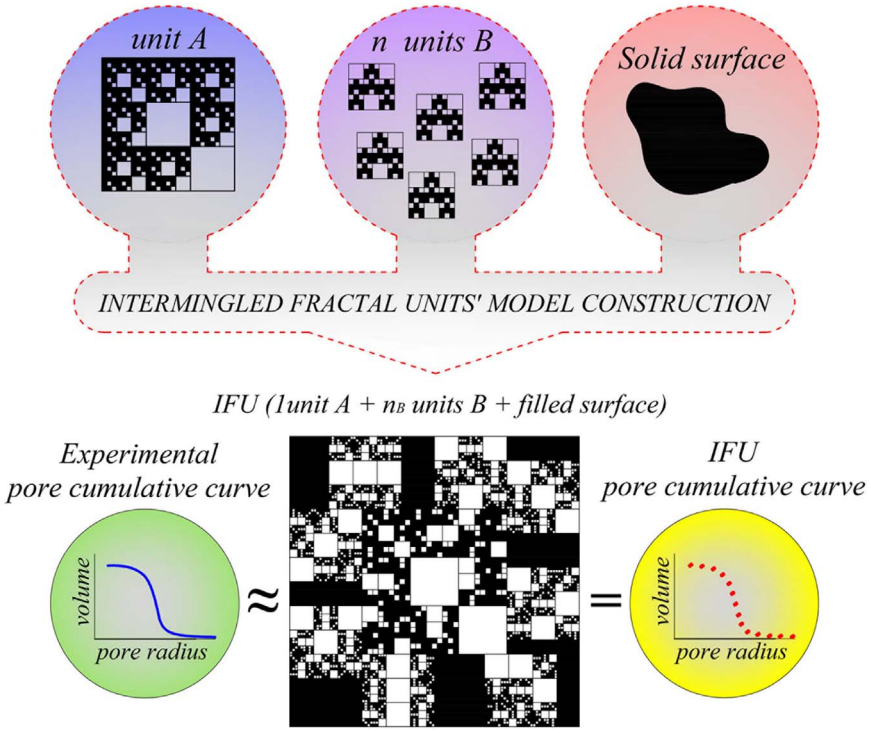

Fig. 1. IFU procedure to simulate experimental pore cumulative curve. Starting point is represented by the geometrical construction of IFU mixing two elementary units based on Sierpinski carpet and a portion of filled surface. The resulting IFU is capable of reproducing the pore cumulative curve, the pore size distribution and the pore volume fraction of experimental porosimetric data.

justifying the definition of macroscopic or 'effective' properties [40]. Effective properties of heterogeneous media are obtained based on direct measurements (experimental and numerical), semi-empirical relations and theoretical techniques. Empirical relations successfully provide effective properties, but they tend to be beneficial in correlating data rather than predicting them.

Clearly, effective properties depend not only on the single phase properties but also on the details of the microstructure (phase volume fractions, orientations, sizes, shapes and spatial distribution of the domains, connectivity of the phases etc.).

Roberts et al. studied the elastic properties of two-phase ceramics composed by solid matrix and voids. In order to analyse the relationship between microstructure and mechanical response, a finite-element method (FEM) capable of taking into account porosity, pore shape and the type of the interconnections between solid regions has been proposed for some microstructures. The reported simple equations allow obtaining a calculated Young's modulus in good agreement with experimental one [41], but they are restricted to monodisperse microstructures and a fixed number of realisations. Numerical analyses can also be combined with X-ray tomography data describing the microstructure [42]; such technique is restricted to a single realisation and poses several issues on the microstructure discretisation.

Atzeni et al. proposed an application of holistic model based on fuzzy math for predicting the mechanical properties of a series of vesicular basalt stones, wherein variables are determined by using porosimetric, mineralogical and weathering data [43].

Semi-empirical and theoretical techniques are capable of providing estimates and bounds of effective properties based on partial statistical information on the microstructure usually involving correlation functions. Classical bounds are deduced from variational principles: we remember here the classical Voigt and Reuss bounds, which, in the isotropic case, can be computed from the weighted arithmetic and harmonic means of the bulk and shear moduli [44,45], while an improved bound based only on the phase volume fraction (two-point correlation function) was given by Hashin and Shtrikman [46] introducing the polarisation fields. We remember that the lower bounds are trivially zero for a two-phase porous composite.

Bounds containing higher-order statistical information on the microstructure have also been given [47–50]; they improve the two-point bounds but are limited in the applications to the availability of such microstructural information.

To adapt the results of formula for dilute composites to the case of finite porosity, the most common approximation is the self-consistent scheme [51,52], which is based on the elastic solution of an inclusion embedded in an unknown effective medium that must be computed implicitly. Alternatively, the differential method (see the review [53]) is based on the computation of the variation of effective properties due to the addition of a small concentration of inclusions. Such two-methods can also be applied to polydisperse heterogeneous media.

This brief overview, on the one hand, shows that porosity has been extensively considered for correlations with elastic behaviour of materials [54,55], but, on the other hand, it highlights that the influence of morphological and phenomenological features of porosity has been often neglected [56,57].

It is clear that the interpretation of porosimetric data has to be done by using good reproduction and representation of microstructures, with particular attention to geometrical and morphological features. In this sense, Euclidean geometry results into a rough instrument for describing complex shape, which can be often recognised in natural or artificial systems. On the other hand, the application of Fractal Geometry theory has aroused great interest and has shown a new road for modelling approach.

Fractal Geometry, as representation method, has its roots long time ago. However, its formalisation dates back to 1975 by Mandelbrot [58]. The central notion of Fractal Geometry is the concept of dimension. Euclidean description identified only integer values. Indeed, in a three-dimensional domain, any space is associated to a characteristic number, called dimension, which describes all points in the considered system. In this sense, the Euclidean dimension is equal to 0 for points, 1 for lines, 2 for surfaces and 3 for volumes. This definition is limited to sets, defined *dimensionally concordant sets*, for which useful dimensions coincide [58]. However, this definition is problematic for subsets, wherein more complex shapes are present. In order to be described, several geometric figures need a non-integer dimension value; the fractal dimension (D_f). One of the most explicit examples, which summarised all properties of fractals, is represented by Sierpinski carpet. It