Optics Communications 420 (2018) 183–188

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Classical-driving-assisted entanglement trapping in photonic-crystal waveguides

Jing Wang *, Yunan Wu, Na Guo, Zhen-Yu Xing, Yue Qin, Pei Wang

School of physics and technology, University of Jinan, Jinan, 250022, China

ARTICLE INFO

Keywords:
Non-Markovianity
Entanglement
Photonic crystal

ABSTRACT

We investigate the exact entanglement dynamics of two classical-driven atoms, each of which is embedded in a single-end photonic crystal waveguide. The finite end of the waveguide behaves as a perfect mirror, forcing part of the emitted light to return back to the atom. The round-trip time between the mirror and the atom is equivalent to the memory time of the open system. It is shown that, the memory time and the classical driving strength are two ingredients whose interplay plays a key role in controlling entanglement. By manipulating the two ingredients, a bound atom–photon state appears in atom–mirror interspace so as to induce entanglement trapping. We also consider the entanglement transfer between different subsystems. We find that, by applying a controllable classical field, the trapped entanglement can be released to the waveguide, hence we can obtain entangled photon pulses directionally. We also discuss a feasible experimental realization of our prediction.

1. Introduction

The effect of the geometric constraints on the interaction between atomic systems and the electromagnetic field has been considered a basic one in the study of quantum electrodynamics (QED). For Cavity-QED [1,2], the emitter couples only to a discrete set of field modes, which leads to the regime of strong atom–field interaction. This strong coupling, characterized by a reversible exchange of excitation between an atom and the mode of cavity, can lead to controllable atom–atom entanglement [3–13] and plays a key role in many quantum information processes [14–21]. Besides the Cavity-QED, a new area of QED is explored in one-dimensional (1D) waveguides, where a small number of atoms couples to an 1D continuum [22]. Nowadays, with the development of technology, the electromagnetic field can be confined within only 1D, and the strong coupling between 1D waveguides and a small number of atoms has been developed [23,24]. These include photonic-crystal waveguide with embedded quantum dots [25], optical microfibers with atoms [26], hollow core fibers [27] or microwave transmission lines coupled to superconducting qubits [28]. These 1D systems provide a broad quantum optical properties, such as population trapping without decay [29], giant Lamb shifts [30], and photon scattering due to the interference of the absorbed and directly transmitted wave [31]. These properties can be used to develop atomic light switches [32], quantum computation [33], entanglement production [34–36], quantum networks [37] and single-photon transistors [38].

In order to further expand the application area of the 1D system, we study the entanglement dynamics and its potential application in this system. We consider two classical-driven atoms, each coupled to a single-end photonic-crystal waveguide. This single-end structure, i.e., the semi-infinite 1D waveguide, can be regarded as an infinite 1D waveguide with a perfect mirror. We choose the semi-infinite 1D waveguide, because the fact that the photonic-crystal waveguide is actually terminated, with the end typically lying on the junction between the waveguide itself and the air medium, imposing a hard-wall boundary condition on the field [25]. In such a 1D structure, the radiation emitted by the embedded atom can be reflected back by the mirror and hence has a significant chance to be reabsorbed by the atom. The feedback behavior may induce the information backflow, i.e., the non-Markovianity [39]. In this non-Markovian open system, the time taken by the emitted photon to perform a round trip between atom and the mirror should behave as the memory time of the system. In this work, we highlight the effect of the memory time and the classical driving strength on the entanglement dynamics between atoms. It originates from the fact that the memory time, which depends on the position of the embedded atom, can be accurately controlled in the experiment [40]. A detailed asymptotic analysis shows that the atom–atom entanglement is strongly related to the memory time as well as the classical driving strength. Under control of the two ingredients, the entanglement can be trapped in the atom–mirror interspace. And, more remarkable, through the simple application of a classical field, the

* Corresponding author.
E-mail address: wynwj_88@163.com (J. Wang).

https://doi.org/10.1016/j.optcom.2018.03.066
Received 18 January 2018; Received in revised form 17 March 2018; Accepted 26 March 2018

trapped atom–atom entanglement can be released to the waveguide as entangled photon pulses. Finally, a possible experimental realization for our illustrated phenomena is discussed.

This paper is organized as follows. The physical model is given in Section 2. In Section 3, the entanglement dynamics of the two-atom system is studied. In Section 4, we study the entanglement transfer between different subsystems. We summarize our results in Section 5.

2. Physical model

The physical model we considered in this work is depicted in Fig. 1, which is created in a planar photonic crystal (PC) platform [41,42]. It consists of two qubits (two-level atoms) and two 1D semi-infinite waveguides along x-axis. The two 1D waveguides a and b, whose end lies at $x = 0$, are coupled, respectively, to qubits A and B at $x = x_0$. The qubits are initially entangled, and both are driven by the classical field with frequency ω_L. Additionally, we assume that the subsystems Aa and Bb are identical and no direct interactions exist between them. By neglecting the counter-rotating terms, the Hamiltonian, for each local subsystem reads ($\hbar = 1$)

$$H = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k a_k^\dagger a_k + \Omega\left(e^{-i\omega_L t}\sigma_+ + H.c.\right) + \sum_k (g_k a_k^+ \sigma_- + H.c.), \quad (1)$$

where ω_0 is the atomic transition frequency, $\sigma_+ = \sigma_-^\dagger = |e\rangle\langle g|$ and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ associated with the atomic excited state $|e\rangle$ and ground state $|g\rangle$; a_k and $a_k^\dagger$ are the annihilation and creation operators for the kth field mode with frequency ω_k; Ω is the coupling constant between qubit and the classical field, which is chosen to be real.

We assume that the waveguide end behaves as a perfect mirror. Thus, part of emitted photon of the qubit will perform a round trip between mirror and the qubit. In the case of a semi-infinite waveguide, the photon dispersion relationship can be linearized around the qubit frequency as [22] $\omega_k = \omega_0 + v(k - k_0)$, where k_0 is the carrier wave vector with $\omega_{k0} = \omega_0$, and v is the photon group velocity. The coupling strength between qubit and the kth mode can be given by [43]

$$g_k = \sqrt{\Gamma v/\pi}\sin kx_0, \quad (2)$$

where Γ is the spontaneous emission rate of the qubit. In the dressed state basis $\{|+\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|e\rangle - |g\rangle)\}$, by using two unitary transformation $U_1 = \exp(-i\omega_L\sigma_z t/2)$ and $U_2 = \exp(-i\omega_0\xi_z t/2)$ [44,45] to the Hamiltonian in Eq. (1), the effective Hamiltonian can be given by

$$H_{eff} = \frac{\omega_{ef}}{2}\xi_z + \sum_k \omega_k a_k^\dagger a_k + \sum_k (G_k a_k^+ \xi_- + H.c.), \quad (3)$$

where $\omega_{ef} = 2\Omega + \omega_0$, $G_k = g_k/2$, $\xi_+ = \xi_-^\dagger = |+\rangle\langle -|$ and $\xi_z = |+\rangle\langle +| - |-\rangle\langle -|$.

We assume that at time $t = 0$, the qubit is in the state $|+\rangle$ and the waveguide in the vacuum states $|\tilde{0}\rangle$. The state vector of the system at any time t, in the interaction picture, is therefore

$$|\varphi(t)\rangle = c_+(t)\left|+,\tilde{0}\right\rangle + \sum_k c_k(t)\left|-,\tilde{1}_k\right\rangle, \quad (4)$$

where the state $\left|\tilde{1}_k\right\rangle$ accounts for the field mode with frequency ω_k having one excitation. By using the Schrőinger equation, the equations for the amplitudes $c_+(t)$ and $c_k(t)$, can be given by

$$\dot{c}_+(t) = -i\sum_k G_k e^{i(\omega_{ef}-\omega_k)t} c_k(t), \quad (5)$$

$$\dot{c}_k(t) = -iG_k c_+(t) e^{-i(\omega_{ef}-\omega_k)t}. \quad (6)$$

By formal time integration of Eq. (6), and eliminating $c_k(t)$ from Eq. (5), we obtain

$$\dot{c}_+(t) = -\int_0^t f\left(t-t'\right) c_+\left(t'\right) dt' \quad (7)$$

where $f\left(t-t'\right) = \sum_k |G_k|^2 e^{i(\omega_{ef}-\omega_k)\left(t-t'\right)}$ is the memory kenel, i.e., the measure of the reservoir's memory of its previous state on the time scale for the evolution of the atomic system. The memory kenel depends strongly on the spectral density of the field. For our model, the spectral density is simply proportional to the square of the atom–photon coupling $|G_k|^2$, and can be expressed as

$$J(\omega-\omega_0) = \frac{\Gamma}{4\pi}\sin^2\left[\frac{t_d}{2}(\omega-\omega_0) + \frac{\phi}{2}\right], \quad (8)$$

where $\phi = 2k_0x_0$ is the optical length of twice the path between the qubit and the mirror, and $t_d = 2x_0/v$ is the finite time taken by a photon to perform a round trip between qubit and the mirror, which behaves as an environmental memory time [43]. Clearly, the width of the spectral density is decided by the memory time t_d, which is different from the Lorentzian spectral density. Exploiting Eqs. (7) and (8), the amplitude $c_+(t)$ can be transformed to

$$\dot{c}_+(t) = -\frac{\Gamma}{8}c_+(t) + \frac{\Gamma}{8}c_+\left(t-t_d\right)e^{i(2\Omega t_d+\phi)}\Theta\left(t-t_d\right), \quad (9)$$

where $\Theta(t)$ is the Heaviside step function. By Laplace transform, we can obtain the Laplace transform $\tilde{c}_+(s)$ for the amplitude $c_+(t)$:

$$\tilde{c}_+(s) = \frac{1}{s + \frac{\Gamma}{8} - \frac{\Gamma}{8}e^{i(2\Omega t_d+\phi)}e^{-st_d}}, \quad (10)$$

By numerically solving the above equation, we can obtain the amplitudes $c_+(t)$.

Clearly, the open dynamics of the system is greatly influenced by the memory time t_d, the phase ϕ and the driving strength Ω, which can be seen from the Eq. (9). When $t \le t_d$, the atom undergoes standard spontaneous emission. After this, the presence of the mirror mainly determines the dynamics of the system. Due to the feedback effect of the mirror, the light emitted in the past can interfere with the light emitted in the present, which is witnessed by the phase factor $e^{i\phi}$ and Ω. In what follows, we study the effect of the memory time t_d, the phase ϕ and the classical driving strength Ω on the entanglement dynamics of the two-qubit system.

3. Entanglement dynamics

The bipartite entanglement can be measured by the concurrence [46], which is defined as

$$C = \max\left\{\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, 0\right\}, \quad (11)$$

where $\{\lambda_i\}$ are the eigenvalues, in the descending order of value, of the matrix $R = \rho_{AB}(\sigma_y^A \otimes \sigma_y^B)\rho_{AB}^*(\sigma_y^A \otimes \sigma_y^B)$, with ρ_{AB}^* denoting the complex conjugate of the density matrix ρ_{AB}.

If the two-qubit system is initially in the entanglement state $|\Psi(0)\rangle_{AB} = \beta|++\rangle_{AB} + \gamma|--\rangle_{AB}$, and the reservoir is in the vacuum state $\left|\tilde{0},\tilde{0}\right\rangle_{ab}$. Exploiting Eq. (4), the reduced density matrix for the two-qubit system can be calculated as (in the basis $\{|1\rangle = |++\rangle, |2\rangle = |+-\rangle, |3\rangle = |-+\rangle, |4\rangle = |--\rangle\}$)

$$\rho_{AB}(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & 0 & 0 \\ 0 & 0 & \rho_{33}(t) & 0 \\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix}, \quad (12)$$

with the density matrix elements evolving as

$$\rho_{11}(t) = \beta^2|c_+(t)|^4,$$

$$\rho_{14}(t) = \rho_{14}^*(t) = \gamma^*\beta|c_+(t)|^2,$$

$$\rho_{22}(t) = \rho_{33}(t) = \beta^2|c_+(t)|^2(1 - |c_+(t)|^2),$$

$$\rho_{44}(t) = 1 - \rho_{11}(t) - \rho_{22}(t) - \rho_{33}(t). \quad (13)$$