Optics Communications 322 (2014) 40–43

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

# Using loss to flat the delay lines of finite coupled-resonator optical waveguides

He Tian [a,*], Weimin Sun [a], Yundong Zhang [b]

[a] Key Lab of In-fiber Integrated Optics, Ministry Education of China, Harbin Engineering University, Harbin 150001, China
[b] National Key Laboratory of Tunable Laser Technology and Institute of Opto-Electronics, Harbin Institute of Technology, Harbin 150080, China

ARTICLE INFO

*Article history:*
Received 2 July 2013
Received in revised form
31 December 2013
Accepted 30 January 2014
Available online 13 February 2014

*Keywords:*
Slow light
Dispersion
Optical storage
Optical device

ABSTRACT

We derive the expressions of transfer function and group delay of finite coupled-resonator optical waveguides (CROWs), using the method of coupling of modes in time, and investigate the influences of loss on the delay line. The results show that loss can eliminate the ripples and sharp peaks in the delay line. A Lorentzian response of the delay line gradually appears with the increase of loss, and the bandwidth is proportional to the amplitude decay rate. Combining with coupling coefficients loss can produce the delay line with a wide flat top where the group velocity dispersion is almost zero. We also experimentally demonstrate this in a fiber CROW which can be utilized for rotation sensing and Mach–Zehnder interferometers. The results can be used to design delay lines for the applications of CROWs.


## 1. Introduction

Coupled-resonator optical waveguides (CROWs), which can be used in optical filters, enhanced nonlinear optical interactions, laser amplifiers, and so on, have been attracting considerable theoretical and experimental attentions [1–3]. Compared with conventional waveguides, one of the most important properties of CROWs is that they display unique and strong dispersive effects because the time that a light wave spends in interacting with the resonators critically depends on the detuning of the light wave from the resonant frequency [4–6]. Thus in CROWs, one can obtain slow light which is considered as a promising approach for the applications requiring the control of delay, such as optical delay lines, dispersion compensations, optical buffers and slow light interferometers [7–14]. However, because of the interactional inhomogeneity between resonators around the resonant frequency, there are always ripples and sharp peaks in the delay lines of finite CROWs, which is detrimental for their applications. In optical communication system, the ripples or peaks result in large group velocity dispersion (GVD) and induce distortions and broadening of optical pulses propagating through the finite CROWs. And in slow light interferometers the sensitivity is proportional to group delay, so the unflatness of the group delay line introduces tremendous instabilities which must be avoided [14–16]. By carefully designing the coupling coefficients, the amplitudes of the ripples or peaks can be reduced to some degree [17], but it is hard to realize determinate coupling coefficients with a very high precision in practice, especially in the case of the CROW consisting of a large number of resonators.

Loss is unavoidable in CROWs, and usually results from bending, absorption and scattering of the material, and coupling to radiated fields. And loss can reduce the transmittance and quality factors. If loss can be utilized to optimize the performance of CROWs, this will doubtlessly boost their applications. In this letter, we derive the expression of group delay, following the theoretical derivations in [18], and investigate the influences of loss on the delay line. Further, we construct a fiber CROW to demonstrate the improvements of the delay line using loss.

## 2. Temporal coupled-mode equations of finite CROWs

A finite CROW consisting of $N$ resonators is shown in Fig. 1, where $a_n$, $\omega_n$ and $R_n$ ($n=1, 2,\ldots,N$) are the energy amplitude, resonant frequency and radius of the $n$th resonator.

The stored energy of light in the $n$th resonators is ($|a_n|^2$). Also energy couples between resonators, and is carried away by the reflected wave $s_r$ and transmitted wave $s_t$. Thus the energy in the resonators leaves through three aspects: coupling to adjacent resonators, coupling to the input/output waveguide, and loss such as absorption and scattering loss. Then we can obtain the temporal coupled-mode equations

$$\frac{d}{dt}a_1=\left(-j\omega_1-\frac{1}{\tau_r}-\frac{1}{\tau_{l,1}}\right)a_1+j\mu_1 a_2+j\mu_r s_i,\ldots,$$

* Corresponding author.
*E-mail address:* tianhe176176@163.com (H. Tian).

http://dx.doi.org/10.1016/j.optcom.2014.01.078
0030-4018 

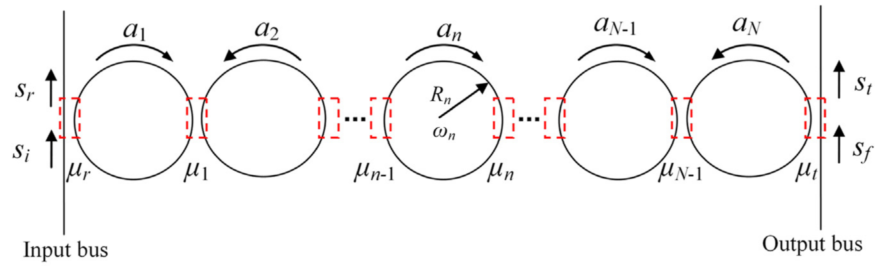


**Fig. 1.** Wave propagation model in a finite CROW. $s_i$ and $s_f$ are the incident and feedback waves, and $s_r$ and $s_t$ are the reflected and transmitted waves. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$\frac{d}{dt}a_{N-1} = \left(-j\omega_{N-1} - \frac{1}{\tau_{l,N-1}}\right)a_{N-1} + j\mu_{N-1}a_N + j\mu_{N-2}a_{N-2},$$
$$\frac{d}{dt}a_N = \left(-j\omega_N - \frac{1}{\tau_t} - \frac{1}{\tau_{l,N}}\right)a_N + j\mu_{N-1}a_{N-1} + j\mu_t s_f, \quad (1)$$

$$s_r = s_i + j\mu_r a_1, \quad (2)$$

$$s_t = s_f + j\mu_t a_N, \quad (3)$$

where $\mu_r$ and $\mu_t$ describe the mutual coupling between resonators and the waveguides (input and output waveguides), and $\mu_n$ describe the mutual coupling between resonators, as shown in Fig. 1. $1/\tau_r$, $1/\tau_t$, $1/\tau_{l,n}$ are the energy amplitude decay rates due to coupling of energy to the waveguides and loss in resonators.

$$\mu_r{}^2 = \tfrac{2}{\tau_r} = k_r{}^2 \tfrac{c}{2\pi R_1 n_e}, \quad \mu_t{}^2 = \tfrac{2}{\tau_t} = k_t{}^2 \tfrac{c}{2\pi R_N n_e},$$
$$\mu_n{}^2 = k_n{}^2 \frac{c^2}{(2\pi n_e)^2 R_n R_{n+1}}, \quad \frac{1}{\tau_{l,n}} = (1-\alpha^2)\frac{c}{2\pi R_n n_e}, \quad (4)$$

where $k_r$, $k_t$, $k_n$ are the coupling coefficients, $n_e$ is the effective index, and $\alpha$ is the attenuation coefficient per circulation in resonators. When $s_i$ is a steady-state signal ($s_i \sim \exp(-j\omega t)$) and there is no feedback ($s_f = 0$), we solve Eq. (1) and then have

$$a_1 = \tfrac{j\mu_r s_i}{\Gamma_1}, \quad a_N = \frac{(j)^N \mu_1\mu_2 \cdots \mu_{N-1}\mu_r s_i}{\prod_{n=1}^{N} \Gamma_n},$$
$$\Gamma_n = \Delta_n + \frac{\mu_n{}^2}{\Delta_{n+1} + (\mu_{n+1}{}^2/(\Delta_{n+2} \cdots + \mu_{N-1}{}^2/\Delta_N))},$$
$$\Delta_1 = -j\Delta\omega_1 + \frac{1}{\tau_{l,1}} + \frac{1}{\tau_r},$$
$$\Delta_n = -j\Delta\omega_n + \frac{1}{\tau_{l,n}}, \quad \Delta_N = -j\Delta\omega_N + \frac{1}{\tau_{l,N}} + \frac{1}{\tau_t}. \quad (5)$$

$\Delta\omega_n = \omega - \omega_n$ is the frequency detuning. The transfer function and group delay can be derived from Eqs. (3) and (5), and are given by

$$T = \frac{s_t}{s_i} = \frac{(j)^{N+1}\mu_1\mu_2 \cdots \mu_{N-1}\mu_r\mu_t}{\prod_{n=1}^{N} \Gamma_n}, \quad (6)$$

$$t_g = \frac{\partial \arg(T)}{\partial \omega}. \quad (7)$$

From Eqs. (6) and (7) it can be seen that the group delay is very sensitive to the frequency of input light and coupling coefficients, so flat spectra of group delay are hardly to be achieved without a very precise control of the coupling coefficients, especially in the case of the CROW consisting of a large number of resonators. Note that Eqs. (6) and (7) can also be used to analyze the filter response of finite CROWs.

Here, we assume that the resonators have the same attenuation coefficient and the same size ($R_n = R$), making that $1/\tau_{l,n} = 1/\tau_l$ and $\Delta\omega_n = \Delta\omega$. It is interesting that when the loss is large ($1/\tau_l \gg \mu_n$, $1/\tau_r$, $1/\tau_t$) we have

$$t_g = N\frac{1/\tau_l}{\Delta\omega^2 + (1/\tau_l)^2}. \quad (8)$$

Thus, the group delay is proportional to the number of resonators $N$, but the bandwidth of the group delay line is independent of $N$. The delay line shows a Lorentzian response, and the full width at half-maximum (FWHM) is $2/\tau_l$.

## 3. Results and discussions

From Eqs. (5)–(7) we can see that the interactional inhomogeneity between resonators can make the delay lines of finite CROWs rippled around the resonant frequency. And the interactional inhomogeneity more easily happens as the increase of the number of resonators composing the finite CROW. Fortunately, loss can reduce the interactional inhomogeneity and further make the delay line not so sensitive to the light frequency and coupling coefficients. Fig. 2 shows the influences of loss on the delay lines and GVD of CROWs consisting of two resonators and four resonators. When there is no loss ($\alpha = 1$), the ripples in the delay lines are apparent and there are sharp peaks in the delay line of the CROW consisting of four resonators. Meanwhile, the GVD is large and also rippled. As the loss increases gradually, the ripples and peaks in the group delay lines are finally eliminated and wide flat tops without GVD (blue lines in Fig. 2) emerge. When the loss is large, the Lorentzian responses of the delay lines appear. Moreover, the loss reduces the time light spends in circulating within each resonator, so the largest delay decreases with the increase of loss.

We also experimentally obtain a delay line with a wide flat top in a fiber coupled-resonator optical waveguide (FCROW), which can be utilized for rotation sensing and Mach–Zehnder interferometers [14–16]. The experimental setup is schematically shown in Fig. 3. The intensity modulator (IM) is used to modulate the laser into Gaussian-shaped pulse whose FWHM is about 50 ns. The polarization controller (PC) is adjusted to excite one of the eigenpolarizations of light in the FCROW. The FCROW which we use in the experiment consists of two fiber ring resonators and the circumference of each resonator is about 50 cm. Meanwhile the 3 dB splitter is used to divide the modulated optical field into two parts: one part is taken as a reference light and the other a signal light. The reference light and signal light are detected by detectors D1 and D2. Therefore, the pulse delay can be measured by comparing the reference light and signal light. The FCROW is immersed in a water box to limit thermal fluctuations.

Fig. 4 shows the experimental results of group delay and transmittance in the FCROW. In the experiment the frequency intervals between experimental points are judged by the transmittance and tuning quantity of laser frequency. The experimental errors are mainly caused by two factors: (1) the slight fluctuations of water temperature and laser frequency, and (2) small deviations of the parameters in the experiment from those in the theoretical