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Efficient optimization of super-oscillatory lens and transfer function analysis in confocal scanning microscopy

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ABSTRACT

Super-oscillatory lens (SOL) provides a promising way to achieve subwavelength focusing in the regime of far-field optics and realize super-resolution imaging in confocal scanning microscopy (CSM). Both binary amplitude and phase SOLs are designed with an efficient optimization method using genetic algorithm and fast Hankel transform algorithm either in oil immersion medium or in air. A much brighter hotspot is readily focused by a phase SOL compared with the amplitude counterpart, e.g., 5.8 times as bright as the latter. To fundamentally interpret the super-resolution imaging mechanism by SOL in CSM, transfer function analysis is conducted compared with basic confocal imaging. The coherent transfer function (CTF) is derived and numerically calculated. The extension of the cutoff frequency and the remarkable enhancement of the magnitude of CTF in the high frequency passband account for super-resolution imaging by SOL in CSM; however, the limited extension of the frequency domain also implies that the attainable resolution is physically limited.

1. Introduction

A variety of novel super-resolution focusing approaches have been proposed to beat Abbe's diffraction resolution limit since 2000, e.g., using superlens [1,2], plasmonic lens [3,4], super-oscillatory lens (SOL) [5,6], high numerical aperture (NA) lens- or mirror-based optical systems with radially polarized vector beams [7–10], in contrast to the conventional pupil filtering engineering method [11–14]. Among these methods, super-resolution focusing by a binary SOL might be most easy to implement [6], and the subwavelength hotspot is created far beyond the near-field region (e.g., tens of wavelengths away from the surface of SOL) [6], thus without contributions from near-field evanescent waves [5]. Super-resolution focusing by SOL is realized by tailoring the interference of a large number of diffracted beams from the nanostructured rings, and in principle there is no theoretical limit on probing an arbitrarily sharp hotspot [5,6]. The highly compressed hotspot readily surpasses the diffraction resolution limit of $\lambda_0/(2\mathrm{NA})$ and practically reaches the known theoretical limit of $0.36\lambda_0/\mathrm{NA}$ under lens- or mirror-based systems in the far-field optics [15–17]. Confocal scanning microscopy (CSM) [18] provides a practical way to realize super-resolution imaging with SOL in the illumination light path, which has been predicted in Ref. [5] and experimentally confirmed in Ref. [6], respectively. So far, subwavelength focusing and super-resolution imaging through SOL are analyzed based on the angular spectrum theory and explained in the spatial domain [5,6]. Transfer function analysis is an essential way to reveal the overall super-resolution imaging mechanism by SOL in CSM [18].

In this paper, an efficient optimization design method is described for various super-resolution SOLs with purposely configured genetic algorithm and fast Hankel transform algorithm, which promises a rapid optimization of SOL within several hundred iterations in contrast to the method reported in Ref. [6]. In addition to binary amplitude SOLs [6], binary phase SOLs are also considered either in oil immersion medium or in air. It is found that a much brighter hotspot is possible to be created by a phase SOL, e.g., 5.8 times as bright as that by an amplitude counterpart. Then, transfer function analysis is presented for fundamental interpretation of the super-resolution imaging mechanism by SOL in CSM. The coherent transfer function is derived and numerically calculated, which is compared with that for the basic confocal imaging. The extension of the cutoff frequency and the remarkable enhancement of the magnitude of CTF in the high frequency passband account for super-resolution imaging by SOL in CSM; however, the limited extension of the frequency passband also implies that the attainable resolution is physically limited, which is in contrast to the claim "The new technology, which in principle has no physical limits on resolution" in Ref. [6]. As a kind of super-resolution microscopy, transfer function analysis on CSM with SOL reflects its fundamental imaging property. The described optimization method is readily

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modified for designing more complex SOLs, e.g., a multi-phase SOL illuminated by a nonuniform incident beam. Transfer function analysis is equally applied to other types of circularly symmetric SOLs in CSM with the coherent illumination beam of arbitrary amplitude distribution.

2. Efficient optimization design of binary amplitude and phase SOLs

2.1. Diffraction propagation based on scalar angular spectrum theory

The scalar angular spectrum theory is a widely used, simple way to analyze many diffraction propagation problems [19], which has been successfully used to design SOLs [5,6]. Let $t(x,y)$ denote the transmission function of SOL at the aperture plane of $z=0$, and the corresponding Fourier spectrum is expressed as

$$T(m,n)=\int\int_{-\infty}^{\infty} t(x,y)\exp[-j2\pi(mx+ny)]dxdy, \tag{1}$$

where m and n are the spatial frequency components in the x and y directions, respectively; suppose $t(x,y)$ is circularly symmetric, and Eq. (1) reduces to the zeroth-order Hankel transform, as

$$T(l)=\int_0^{\infty} t(r)J_0(2\pi lr)2\pi r dr, \tag{2}$$

where $l=(m^2+n^2)^{1/2}$, $r=(x^2+y^2)^{1/2}$, and $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind. Suppose a uniform plane wave of unit amplitude is perpendicularly incident upon SOL, the light field in the observation plane at a distance of z away from the surface of SOL is determined, according to the scalar angular spectrum theory [19], by

$$U(x,y,z)=\int\int_{-\infty}^{\infty} T(m,n)\exp\{j2\pi[mx+ny+q(m,n)z]\}dmdn, \tag{3}$$

with $q(m,n)=(1/\lambda^2-m^2-n^2)^{1/2}$; $T(m,n)$ satisfies circular symmetry, and thus Eq. (3) reduces to

$$U(r,z)=\int_0^{\infty} T(l)\exp[j2\pi q(l)z]J_0(2\pi lr)2\pi l dl, \tag{4}$$

with $q(l)=(1/\lambda^2-l^2)^{1/2}$; again, $U(r,z)$ is expressed as a zeroth-order Hankel transform. If immersing SOL into a liquid medium with the refractive index being η, the wavelength λ in Eqs. (3) and (4) should be replaced by $\lambda=\lambda_0/\eta$ with λ_0 being the vacuum wavelength. Both binary amplitude and phase SOLs are designed either in oil immersion medium ($\eta\approx 1.515$) or in air ($\eta\approx 1$).

2.2. Efficient optimization with genetic algorithm and fast Hankel transform algorithm

From Eqs. (2) and (4), the intensity distribution, $I(r,z)=|U(r,z)|^2$, is determined by two zeroth-order Hankel transforms; in order to characterize a required subwavelength hotspot (or a star-halo disc), similar to those reports [5,6,20], a constrained, single-objective optimization model is established as

$$\begin{aligned}
&\min \quad \frac{I(d_0/2,z;t_i)}{I(0,z;t_i)}\\
&s.t. \quad \frac{I(r,z;t_i)}{I(0,z;t_i)}\le 0.2,\quad d_0\le r\le \kappa d_0\\
&\qquad t_i\in\{0,1\}\ \ or\ \ \{-1,1\},\quad i=1,2,\ldots,N\\
&\qquad 0.60\le\ \sin\left[tan^{-1}\left(\frac{R}{z}\right)\right]\le 0.90
\end{aligned} \tag{5}$$

where d_0 restrains the full-width-at-half-maximum (FWHM) of the hotspot (central main lobe); the radial width of the transition dark region between the central hotspot and the surrounding large halos is set to be $(\kappa-1)d_0$, among which the normalization maximum intensity is constrained to be no larger than 20% of the peak intensity of the central main lobe; for a binary phase SOL, the transmission of the ith ring $t_i\in\{-1,1\}$; N is the total number of the concentric rings contained in the SOL; R is the radius of the SOL; the location of the optimal hotspot is constrained to be within a finite distance away from SOL, which is far beyond the near-field region, in contrast to the method by surface plasmonic lenses [3,4]. Let z_o denote the distance at which the optimal SOL is reached. For a feasible restriction of z_o, it is constrained within an equivalent numerical aperture between 0.60 and 0.90, and it is found through many calculations that such a restriction is quite useful to rapidly reach a good SOL. The above nonlinear multivariable optimization model particularly requires global optimization algorithms, such as a variety of stochastic optimization algorithms, e.g., simulated annealing algorithm [14], genetic algorithm (GA) [21], and particle swarm algorithm [6].

Genetic algorithm (GA) is elected for solving the above optimization problem, due to flexible choice of many GA variants, easy to implement, convenience of straightforward binary coding, and parallel searching capability [21]. Both binary amplitude and phase SOLs are coded into a series of binary digits {0,1}; as for the phase SOL, the decoding strategy is converting {0,1} into $\{1,-1\}$ via $(-1)^{\{0,1\}}$, i.e., using ‘0′ to represent ‘1’ and ‘1’ for ‘-1’; for the distance z, it is coded using twelve binary digits, with an axial resolution from several nanometers to tens of nanometers depending on the axial searching range; the total length of the binary digits for each individual is hence $(N+12)$. For a stable and fast convergence, GA is configured using two-point crossover, two-point mutation, and the elite selection strategy; GA is set to hold a population of 40–60, with a crossover probability of 0.8, and a mutation probability of 0.12. For each iteration, two zeroth-order Hankel transforms are implemented. In order to substantially accelerate the calculation of Hankel transform, a fast Hankel transform algorithm [22] is programmed to operate with efficient computation and good accuracy especially useful for a large SOL with narrow ring width. The key point of this algorithm is converting an M-point discrete approximation of Hankel transform into a procedure of computing two 2M-term fast Fourier transforms and their multiplication [22]. Specifically, in the following optimization, an illumination wavelength of $\lambda_0=532$ nm is used. Using the above optimization method, it is found by many design examples that the number of iterations within several hundreds is sufficient to reach a good SOL. During each iteration, fast Hankel transform algorithm promises an efficient evaluation of all individuals contained in the current generation, with typically several seconds. The stochastic parallel searching capability of GA and the efficient computation of Hankel transform give rise to a general rapid optimization design of various superresolution SOLs. For comparison, in the following examples, SOL is set with a total ring number of $N=100$ and an equally spaced ring width of 200 nm, and the diameter of such a SOL is 40 μm ($R=20$ μm); these initial settings are the same with the example in Ref. [6].

Three examples are provided to show that the highly compressed hotspot is created by binary amplitude and phase SOLs far beyond the near-field region either in air or in oil immersion medium, respectively. The optimized SOLs are tabulated in Table 1. SOL_1 is a binary amplitude element, and SOL_2 is a binary phase one, both designed in air, while SOL_3 is a binary amplitude one designed in oil. In order to describe a SOL more concisely (for displaying a large number of concentric rings), the transmission t_i from the first ring (innermost) to the Nth ring (outermost) is coded by continuously converting every four successive binary digits into one hexadecimal digit; the coded string of hexadecimal digits is shown in Table 1. For example, in the second row of Table 1 (SOL_1), the transmissions of