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Smoothed half-infinite plane waves: Approaching to their optimum profiles

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ABSTRACT

A concept of homogeneous, smoothed half-infinite plane waves is developed in the framework of a discontinuity-free decomposition of the field of a plane electromagnetic wave diffracted by a perfectly conducting half-infinite screen. It is shown that the entire diffracted field is broken down into the mentioned reflected and transmitted, smoothed half-infinite plane waves and edge quasi-cylindrical waves. In the planes of half-waists, the wavefronts of the smoothed waves are always rigorously plane, whereas their amplitude profiles are odd symmetrical in relation to respective half levels. The smoothed waves possess the phase-conjugate property relative to the planes of their half-waists and, in the first approximation, they are self-similar in the entire space. Also, the amplitude profiles at the half-waists of these waves are well reproduced within certain propagation distances. These and other properties of the smoothed half-infinite plane waves, a procedure for their approximate generation, and two simplest analytic profiles at their half-waists are considered in detail.

1. Introduction

The diffraction of a plane monochromatic electromagnetic wave by a half-plane screen with a sharp straight edge is one of a few canonical diffraction problems, of which solutions are known in rigorous forms. In 1896, Sommerfeld was managed to solve this problem [1] on the basis of Maxwell's equations and simplifying assumptions on the infinitely small thickness of the screen and its infinite conductivity. However, the exact integral forms of the resultant diffracted field have been obtained through the angular spectrum approach or, in other treatment, using the Huygens model of diffraction with summing up infinitely many transmitted and reflected plane waves diffracted by the screen. These integral forms give no way of explaining the effect of regular fringing of a diffraction field pattern in the illuminated region behind the screen and its smooth decay in the shadow region. A key to understanding this strange situation was found by Sommerfeld in adopting the Young model of diffraction phenomena: Using another integral representation, the diffracted field around the screen edge was decomposed approximately into two components—a half-infinite geometrical wave and a boundary- or edge-diffracted wave diverged from the edge of the screen. An experimental argument in favour of such decomposition was the fact that the edge of the illuminated screen appears luminous when viewed from the shadow region (the phenomenon known since the time of Grimaldi and Newton).

At the same time, it was not feasible to use the obtained approximate decomposition near the geometrical shadow boundaries of the reflected and transmitted waves, where the error of approximation was increased rapidly and the magnitudes of the geometrical and edge waves experienced discontinuities. Using the same Young model and the concept of Maggi and Rubinowicz [2,3] on transition from the surface diffraction integral to the 1D contour one, taken over the edge of the screen, Ganci has derived the exact integral form of the transmitted edge wave which complements the geometrical wave up to the entire wave field [4,5]. However, as before, the magnitude of the refined edge wave remained discontinuous at the geometrical shadow boundary. For this reason, all the mentioned discontinuous edge waves as did the geometrical waves did not obey the propagation wave equation and thus they were not the real waves. They could be considered in combination only and served for convenience of mathematical calculations and a simplified explanation of observed interference patterns. On the other hand, a number of experimental evidence has been advanced explicitly for the existence of some real modifications of the discontinuous geometrical and edge waves [6–10].

In an attempt to validate the mentioned experimental results and to find new decompositions of the diffracted field, in this paper, we derive the approximate forms of physically realistic, discontinuity-free counterparts of the former discontinuous geometrical and edge waves. The new waves are obtained on the basis of the Sommerfeld diffraction theory, the revised Young model of

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diffraction, and optimization techniques. Considering a finite spatial region with a specified propagation distance, we concentrate, mainly, on the extraction of smoothed, half-infinite plane (SHIP) waves replacing a geometrical wave. This is motivated by the fact that such decompositions of the diffracted field might be useful not only for a new treatment of diffraction phenomena, but also for constructing the profiles of the so-called flat-topped beams with a large size of their uniform central zone. Making a simple generalization of the Prosser′s approach on the problem of diffraction by a slit [11], the total field of similar 2D beams can really be produced by summing up two SHIP waves. The search of optimum profiles of such waves is reduced, actually, to a problem of optimum apodization of their angular spectrum of plane waves.

We emphasize that the intensity profile in the transverse waists of the flat-topped beams is often shaped to the apodized super-Gaussian distribution [12], both for rectangular- and circular-shaped beams. Other numerous versions include the super-Lorentzian [13], Fermi-Dirac [14], cosine-Gaussian [15], hyperbolic-cosine Gaussian [15], Hermite-sine-Gaussian [16], and Hermite-cosine-Gaussian [16] profiles. Also proposed are the profiles in the form of the weighted superpositions of the Hermite–Gaussian [17] and Laguerre–Gaussian [18] beams and that of the fundamental Gaussian beams with varying [19] and constant [20] waists. However, in spite of a large number of proposed profiles, the search of their new kinds is still an urgent problem, since the variations of the shape of these beams can differ substantially within a specified range from their waist.

2. Rigorous and approximate Sommerfeld′s solutions

We begin by recalling that starting from the linearity of Maxwell′s equations Sommerfeld has reduced the vector problem of diffraction of a plane wave by the above screen to a decomposition of the scattered field into two components with different polarizations. Next, he analyzed a particular 2D case when a considered field component, say the electric strength vector **E**, is completely independent of one Cartesian coordinate, say y. The geometry of that problem is reproduced in Fig. 1(a), where a common origin 0 of the Cartesian and cylindrical systems, (x, y, z) and $(r = \sqrt{x^2 + z^2}, y, \vartheta)$, is placed at the edge of screen S

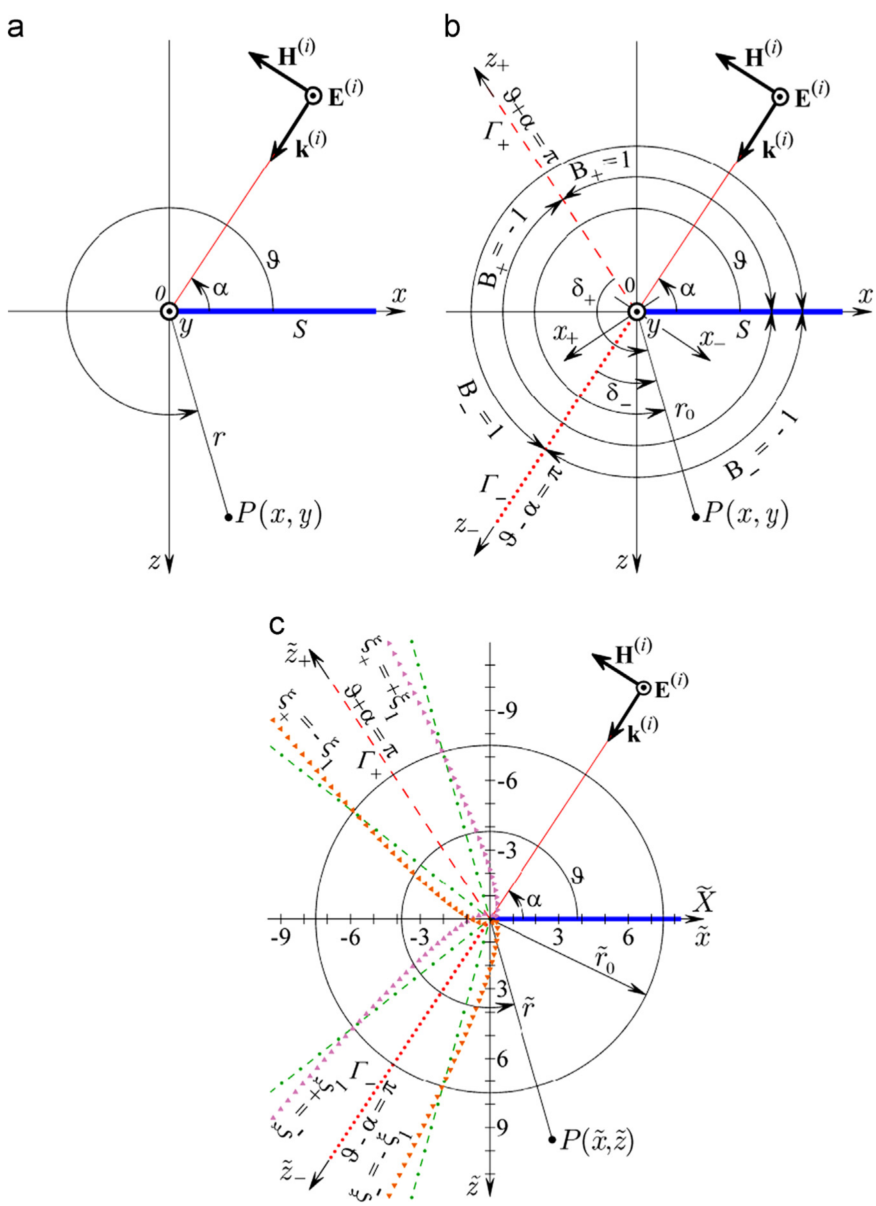

Fig. 1. Initial geometry of the problem (a), the introduction of auxiliary notations (b), and the isolation of the paraxial regions (c).