Physica E 57 (2014) 35–41

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

New aspects of the interaction between two atoms and nonlinear optical fields ☆

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HIGHLIGHTS

- Interaction between moving two atoms and nonlinear optical field (deformed field).
- Entanglement using concurrence and negativity measures for a class of special cases of a two-qubit system.
- Entanglement sudden death (ESD) and entanglement sudden birth (ESB).
- Effects of the different parameters on the entanglement measures.

ARTICLE INFO

Article history:
Received 19 April 2013
Received in revised form
21 September 2013
Accepted 27 September 2013
Available online 7 November 2013

Keywords:
Entanglement
Concurrence
Negativity
Deformed coherent state

ABSTRACT

We present a new kind of interaction between two two-level atoms and optical field initially in deformed bosonic coherent states. Using the concurrence and negativity as measures of entanglement, we investigate the nonlocal correlation between atom–atom and atom–field in terms of the parameters involved in the whole system. We report some important results related to this new kind of interaction such as sudden death, sudden birth, and entanglement stabilization.

1. Introduction

Entanglement is some kind of correlations between two or more quantum systems [1]. The nonlocal nature of entanglement has also been identified as an essential resource for many novel tasks in quantum information processing such as quantum teleportation [2], superdense coding [3], quantum cryptography [4,5] and quantum metrology [6]. These quantum-information tasks cannot be carried out by classical resources and they rely on entangled states. This recognition led to an intensive search for mathematical tools that would enable a proper quantification of this resource. In particular, it is of primary importance to test whether a given quantum state is separable or entangled.

Different entanglement measures and quantifiers have been used for the pure and mixed states such as concurrence [6,7], entanglement of formation [8], negativity [9,10] and Fisher information [11–13]. In this way, the concurrence and negativity are used as good entanglement measures for a mixed state, the von Neumann entropy has been proposed for pure state entanglement [14], all these measures are used to test whether a given quantum state is separable or entangled. Also, some interesting physical phenomena are observed as a result of entanglement measure, such as entanglement sudden death (ESD) and entanglement sudden birth (ESB) [15].

Over the last two decades much attention has been focused on the properties of the Jaynes Cummings model (JCM) for moving an atom. Some theoretical efforts have been stimulated by experimental progress in cavity QED. In addition to the experimental drive, there also exists a theoretical motivation to include the atomic motion effect to JCM because its dynamics become more interesting. Considering the motion of the atoms, the TJCM model with two moving atoms has

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http://dx.doi.org/10.1016/j.physe.2013.09.013

been investigated [16] and the authors have shown that the ESD and ESB [17] have also been experimentally observed for entangled photon pairs [18] and the atom ensembles [19], in addition the entanglement of two moving atoms interacting with a single-mode field via a three-photon process is also investigated [20].

Quantum groups (QGs) have been introduced as a natural extension of the notion of coherent states and interpreted as nonlinear harmonic oscillators with a very specific type of the linearity [21,22]. A q-deformed harmonic oscillator was defined in terms of q-boson annihilation and creation operators, the latter satisfying the quantum Heisenberg–Weyl algebra [23] which plays an important role in QG. The deformed coherent states have been used to describe a large class of quantum systems prepared from several potentials (by a proper choice of the q-deformed parameter) such as infinite, modified Pöschl–Teller, Morse potentials [24] and from the finite range potentials [25].

Over the last two decades, there have been several experimental demonstrations of nonclassical effects. Some important physical concepts of corresponding coherent states exhibit many nonclassical properties such as photon antibunching, sub-Poissonian photon statistics and squeezing (for a review see Ref. [26]). It has been experimentally observed that the real laser, bunched and antibunched light possesses a photon number statistics which can be super-Poissonian or sub-Poissonian. The physical importance of the deformed coherent states lies in the fact that they offer the best description for non-ideal physical devices such as lasers [27] (i.e. real lasers). The deformation parameter plays then the role of a tuning parameter defining how far the realized device is from the ideal one.

Here, we are going to investigate the interaction between two identical or symmetric two-level atom and nonlinear deformed field in the rotating wave approximation. We examine the effects of the deformation and atomic motion parameter on the dynamical properties of the von Neumann entropy and concurrence. Our main goal in this case is to answer the question "Do these parameters have a real effect on the entanglement between two atoms and nonlinear deformed field?"

The paper is organized as follows: in Section 2, we present the model of the nonlinear deformed field and moving two atoms and calculate the atomic density matrix. In Section 3, we present the numerical results and discuss the different effects on the dynamics of the system entanglement. Finally, we summarize the main results in Section 4.

2. Model and its dynamics

In this section, we consider the model of the interaction between a nonlinear deformed field F and symmetric moving two two-level atoms A, B with energy levels denoted by $|+\rangle_j$ and $|-\rangle_j$, where $|-\rangle$ is the lower level and $|+\rangle$ is the upper level of jth two-level atom i.e. j=A, B atom. The interaction Hamiltonian $\hat{H}_I$ of the system in the rotating-wave approximation (RWA) can be written as

$$\hat{H}_I = g(t) \sum_{j=1}^{2} (\hat{a}_q \hat{S}_-^{(j)} + \hat{a}_q \hat{S}_+^{(j)}), \quad \hbar = 1 \tag{1}$$

where $\hat{a}_q$ ($\hat{a}_q^\dagger$) is the annihilation (creation) operator of the deformed field mode. The operators $\hat{S}_+^{(j)} (\hat{S}_-^{(j)})$ and $\hat{S}_z^{(j)}$ are the usual raising (lowering) and inversion operators for jth two-level atom, respectively.

We deal with the one-dimensional case of atomic motion of the cavity axis and denote by $g(t)$ the shape function of the cavity field mode [28,29]. A realization of particular interest is $g(t) = (p\pi v t/L)$ in the presence of atomic motion i.e. $p \neq 0$, and $g(t) = \lambda$ in the absence of atomic motion $p \neq 0$, where v denotes the atomic motion velocity and p stands for the number of half wavelengths of the mode in the cavity. We restrict our study for the atomic motion for the cavity length L along the z-direction. Also, we consider the atomic motion velocity as $v = \lambda L/\pi$ which leads to

$$g_1(t) = \int_0^t g(\tau)\, d\tau = \begin{cases} \dfrac{1}{p}[1 - \cos(p\lambda t)] & \text{for } p \neq 0 \\ \lambda t & \text{for } p = 0 \end{cases} \tag{2}$$

Now, let us assume that the two atoms are initially in the upper state (i.e. $\theta = 0$) and maximally entangled quantum state or *Bell state* (i.e. $\theta = \pi/4$). While the field in the deformed bosonic coherent states $|\alpha\rangle_q$

$$|\Psi_{ABF}(0)\rangle = (\cos\,\theta|+,+\rangle + \sin\,\theta|-,-\rangle) \otimes |\alpha\rangle_q, \tag{3}$$

here $|\alpha\rangle_q$ is the deformed bosonic coherent states are coherent states that are constructed using a formally analogous scheme as the one allowing the construction of the Glauber coherent states starting from the Heisenberg–Weyl algebra. These states are defined as the eigenstate of the annihilation operator of a q-deformed bosonic field $\hat{a}_q$

$$|\alpha\rangle_q = \frac{1}{\sqrt{\exp_q[\alpha^2]}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{[n]_q!}} |n\rangle, \tag{4}$$

where, we have considered $q \in \mathbb{R}$, and the deformed $\exp_q$ is defined as

$$\exp_q[x] = \sum_{n=0}^{\infty} \frac{x^n}{[n]_q!}. \tag{5}$$

The function $\exp_q$ is a deformation version of the usual exponential function. They become coincident when q is the identity. Notice that $\exp_q[x]\exp_q[y] \neq \exp_q[x+y]$ and $[\exp_q[x]]^a \neq \exp_q[ax]$, i.e. we have a non-extensive exponential which can be found in many physical problems [31,32].

In this paper we assume that the deformation can be achieved using the following function [30]:

$$[n]_q = \frac{q^n - q^{-n}}{q - q^{-1}}, \tag{6}$$

and the q-factorial is defined as

$$[n]_q! = [n]_q [n-1]_q \cdots [1]_q; \quad [0]_q! = 1. \tag{7}$$

The wave function $|\Psi(t)\rangle$ at any time $t > 0$ takes the form

$$\begin{aligned} |\Psi(t)\rangle = \sum_{n=0}^{\infty} & \psi_1(n,t)|+,+\rangle|n\rangle + \psi_2(n,t)|+,-\rangle|n+1\rangle \\ & + \psi_3(n,t)|-,+\rangle|n+1\rangle + \psi_4(n,t)|-,-\rangle|n+2\rangle. \end{aligned} \tag{8}$$

The coefficients $\psi_j(n,t)$, j=1,2,3,4, are obtained from solving the Schrödinger equation $(i\hbar\partial|\Psi(t)\rangle/\partial t = \hat{H}_I|\Psi(t)\rangle)$, where $\hat{H}_I$ is given by Eq. (1). The explicit expressions for these coefficients in the classical limit $q \to 1$ are given by

$$\begin{aligned} \psi_1(n,t) = \frac{Q_n}{n!(n+2)! + [(n+1)!]^2} \Big\{ & \cos\,\theta[n!(n+2) \\ & + [(n+1)!]^2 \cos(g(t)\sqrt{4n+6})] \\ & + (n+1)!\sqrt{n!(n+2)!}\sin\,\theta[\cos(g(t)\sqrt{4n+6}) - 1] \Big\}, \end{aligned}$$

$$\begin{aligned} \psi_2(n,t) &= \psi_3(n,t) \\ &= \frac{-iQ_n}{\sqrt{4n+6}} \sin(g(t)\sqrt{4n+6}) \Big\{ \sqrt{n+1}\cos\,\theta + \sqrt{n+2}\sin\,\theta \Big\}, \end{aligned}$$

$$\begin{aligned} \psi_4(n,t) = \frac{Q_n}{n!(n+2)! + [(n+1)!]^2} \Big\{ & \sin\,\theta[n!(n+2)!\cos(g(t)\sqrt{4n+6}) \\ & + [(n+1)!]^2] \\ & + (n+1)!\sqrt{n!(n+2)!}\cos\,\theta[\cos(g(t)\sqrt{4n+6}) - 1] \Big\}. \end{aligned}$$

The atomic density matrix $\hat{\rho}_{AB}(t)$ can be written as follows:

$$\hat{\rho}_{AB}(t) = \mathrm{Tr}_F(|\Psi(t)\rangle\langle\Psi(t)|), \tag{9}$$