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Higher-order Peregrine combs and Peregrine walls for the variable-coefficient Lenells-Fokas equation

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ABSTRACT

In this paper, we study the variable-coefficient Lenells-Fokas (LF) model. Under large periodic modulations in the variable coefficients of the LF model, the generalized Akhmediev breathers develop into the breather multiple births (BMBs) from which we obtain the Peregrine combs (PCs). The PCs can be considered as the limiting case of the BMBs and be transformed into the Peregrine walls (PWs) with a specific amplitude of periodic modulation. We further investigate the spatiotemporal characteristics of the PCs and PWs analytically. Based on the second-order breather and rogue-wave solutions, we derive the corresponding higher-order structures (higher-order PCs and PWs) under proper periodic modulations. What is particularly noteworthy is that the second-order PC can be converted into the Peregrine pyramid which exhibits the higher amplitude and thickness. Our results could be helpful for the design of experiments in the optical fiber communications.

1. Introduction

As one type of nonlinear waves, breathers have recently drawn much attention since they could be considered as potential prototypes for the rogue waves (RWs) in the ocean [1–9] and other fields of physics [10–12]. Breathers develop owing to the instability of small amplitude perturbations that may grow in size to disastrous proportions [13]. Generally speaking, there are two kinds of breathers, namely, the Kuznetsov-Ma breathers (KMBs) that are periodic in space and localized in time [14] and Akhmediev breathers (ABs) that are periodic in time and localized in space [15]. When the period of the breathers tends to infinity, the Peregrine soliton (PS) localized both in time and space is formed, which could be used as the mathematical description of RW [16–19]. RWs also appear in various fields, including the oceanography [18], nonlinear fiber optics [20–24], Bose– Einstein condensates [25–27], atmospheric dynamics [28], plasma [29], laser– plasma interactions [30], and even finance [31], to name a few. They are the localized structures with the instability and unpredictability [32,33], and are short lived and particularly rare walls with devastating effects. The RWs have a peak amplitude generally more than twice the significant wave height and are considered to be waves appearing from nowhere and disappearing without a trace [33].

In optical communications, there always exist some nonuniformities due to various factors, which include the imperfection of manufacture, variation in the lattice parameters of the fiber media and fluctuation of the fiber diameters [34]. Those

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nonuniformities often lead to such effects as the fiber gain or loss, phase modulation, and variable dispersion [35]. The inclusion of the variable coefficients into the nonlinear Schrödinger (NLS) equations is currently an effective way to reflect the inhomogeneous effects of the nonlinear optical pulses [36]. Compared to the nonlinear waves in constant-coefficient models, the breathers and RWs with variable coefficients often show many novel properties such as the nonlinear tunneling effect, breather evolution, amplification, and compression, Talbot-like effects, and composite rogue-wave structures [37–40]. In particular, Tiofack *et al.* have recently investigated the Peregrine comb (PC) generation using multiple compression points of PS in periodically modulated NLS equation [41]. The numerical result shows that the main properties of the PCs persist in nonintegrable situations [41]. Their predictions are in good agreement with numerical findings for an interesting specific case of an experimentally realizable periodically dispersion modulated photonic crystal fiber [41]. Wang *et al.* have further found the Peregrine wall (PW) in the variable-coefficient Hirota equation and studied the effects of the higher-order terms on the PCs and PWs [42]. The third-order dispersion and nonlinearity terms can affect the characteristics of the spatial and temporal distribution of the PCs and PWs [42].

We will focus on a variable-coefficient Lenells-Fokas (LF) equation [43,44]

$$q_{xt} - \alpha(t)\, q_{xx} + 4\,\beta(t)\, q - 2\, i\, \gamma(t)\, q_x \pm 4\, i\, \beta(t) \Big| q \Big|^2 q_x = 0\,, \tag{1}$$

which is an integrable generalization of LF equation involving three arbitrary time-dependent coefficients $\alpha(t)$, $\beta(t)$ and $\gamma(t)$. Note that Eq. (1) recovers the corresponding autonomous LF equation by taking $\alpha(t) = \alpha_0$, $\beta(t) = \beta_0$ and $\gamma(t) = \gamma_0$ with α_0, β_0 and γ_0 as constants. Kundu has shown that Eq. (1) shares the accelerating soliton solution and other unusual features [43] for the time-dependent coefficients $\beta(t) = a_0\, t$ and $\gamma(t) = c_0\, t$. Lü has obtained soliton solutions and investigated the nonautonomous motion of the accelerated and decelerated solitons for Eq. (1) by the bilinear method [44] and has presented the multi-soliton solutions of the autonomous LF equation [45] as well. He *et al.* have presented an analytical representation of the RWs of the autonomous LF equation by means of Darboux transformation (DT) [46,47]. Wang *et al.* have studied the characteristics of the nonautonomous first-order RW and breathers for Eq. (1) [48] as well as the higher-order localized wave structures [49]. However, to our knowledge, the PC and PW structures of Eq. (1) have not been studied elsewhere. In particular, the higher-order PCs and PWs have yet not been reported in previous models [41,42].

In this paper, firstly, we display three types of multiple compression points structures under suitable periodic modulation, including the breather multiple births (BMBs), PCs and PWs. Then we investigate the properties of the PCs and PWs. We finally present higher-order solutions such as the second-order PCs and Peregrine pyramids (PPs). It is expected that the results obtained in this paper will be useful to find the changeable but feasible breathers and RWs in experimentally controlled environments.

The arrangement of the paper is as follows: In Sec. 2, we will construct the generalized ABs solution and present different types of BMBs for Eq. (1). In Sec. 3, we will study the dynamics of the PC and PW structures as well as their spatiotemporal characteristics. In addition, we will analyze the effects of variable coefficient $\alpha(t)$ on these waves. In Sec. 4. we will introduce some physical quantities to further explore the typical characteristics of the PCs. Higher-order solutions of Eq. (1) will be given in Sec. 5. Finally, Sec. 6 will be the conclusions of this paper.

2. Generalized AB solution and BMBs

To construct the generalized AB solution, we consider the following plane-wave solution as the seed solution

$$q^{[0]} = c\, e^{i\,\rho}, \tag{2}$$

where

$$\rho = b(t) + a\, x\,, \quad b(t) = \int \left(\left(\frac{4}{a} - 4\, c^2 \right) \beta(t) + a\, \alpha(t) + 2\, \gamma(t) \right) \mathrm{d}t. \tag{3}$$

The parameter c is the initial amplitude of the background and a is the wave number. Further, by means of the DT of Eq. (1) [48,49], we present the first-order generalized AB solution as follows

$$q^{[1]}_{AB} = \left(c - \frac{2\, i}{n}\, \frac{G^{[1]}_{AB} - i\, H^{[1]}_{AB}}{D^{[1]}_{AB} + i\, F^{[1]}_{AB}} \right) e^{i\,\rho}, \tag{4}$$

with

$$G^{[1]}_{B} = s_1 \cosh(t\, V_H + x\, h_R) + s_2 \cos(t\, V_T + x\, h_I) + s_2^* \sin(t\, V_T + x\, h_I) + s_1^* \sinh(t\, V_H + x\, h_R)\,,$$

$$H^{[1]}_{B} = x_1 \cosh(t\, V_H + x\, h_R) + x_2 \cos(t\, V_T + x\, h_I) + x_2^* \sin(t\, V_T + x\, h_I) + x_1^* \sinh(t\, V_H + x\, h_R),$$