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Heat flow into periodic array of interface cracks

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ABSTRACT

A bonded bimaterial consisting of two homogeneous, dissimilar materials is considered. A periodic array of imperfections in the bond along the interface disturb uniform heat flux in the far field, and result in residual thermal stresses. Stresses, stress intensity factors, and crack opening displacements are determined for two different cases of relative distortivity of the constituent materials.

1. Introduction

Composite materials are frequently used for engineering applications, and their durability is of mathematical interest. Models are often formulated by considering two isotropic materials with dissimilar elastic and thermal properties that are bonded together along an interface. The interface may also be subject to defects such as cracking, which can disturb the uniform, far-field temperature and stress distributions.

Previous research on interface cracks has frequently considered isolated cracks at the interface of composite materials, where loads were specified on the crack and/or at infinity. An isolated interface crack was considered by England (1965). The stresses and displacements were written in terms of complex potentials (Sadd, 2009), and analytic continuation was used to connect the elastic potentials of the two half-spaces. A Riemann–Hilbert problem was derived from the given tractions on the crack, and then solved with techniques from analytic function theory (England, 2003; Muskhelishvili, 1977). The solution determined the stress and displacement derivatives for the whole bimaterial. The solution included an inverse-square-root singularity in the stresses and the displacement derivatives near the crack tips, and the stresses and displacements also oscillated near the crack tips, in a zone of significantly smaller extent than the crack length.

This interface crack problem was reconsidered (Comninou, 1977) by assuming that near the crack tip, a transition zone exists where the crack remains closed and is in frictionless contact. This assumption eliminated the oscillating singularity near the crack tip, but significantly changed the singular behavior of the stresses there. Similar to the work of England (1965), the problem was isothermal. The analysis used a superposition of dislocations to formulate the problem as a singular integral equation.

By using the technique of analytic continuation, a two-phase material can be treated analytically as if it were a single material. It has been shown by Dundurs (1967, 1969) that an isothermal interface crack problem can be described by two dimensionless ratios of elastic constants, instead of the four that would normally be required. Hutchinson et al. (1987) also used the reduction-of-parameters technique for an isothermal problem.

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Thermal loading acts as an eigenstrain (equivalently, a body force) in the elasticity problem (Mura, 1987), and can be considered in addition to or instead of mechanical loading. In the special case of uncoupled thermoelasticity, the temperature distribution can be found independently of the elastic field. In the case of steady-state heat flow, the temperature distribution is governed by Laplace's equation. Florence and Goodier (1960) solved the thermoelastic problem of an ovaloid hole in a homogeneous, isotropic material. The full temperature distribution was found by assuming a uniform flux at infinity, and an insulated hole. In this problem, there was no external mechanical loading. The disturbed heat flux induced dislocations and tractions on the surface of the hole. Stresses in the whole plane were derived by superposing a stress field that annulled these dislocations and tractions. Sadd (2009) also considered the problem for a circular hole and generalized it to an elliptical hole.

When considering interface crack problems, the normal heat flux and temperature are continuous across the bond. The cracks and defects are assumed to be at least partially thermally insulated, which implies a prescribed normal heat flux there. Several authors have used analytic function theory with analytic continuation to solve the uncoupled thermal problem for an interface crack (Brown and Erdogan, 1968; Lee and Park, 1995). Both results indicate that the uniform heat flux is perturbed by a reciprocal-square root term with no oscillations near the crack tips.

After finding the temperature distribution, several different methods can be used to solve the thermoelastic interface crack problem. Brown and Erdogan (1968) solved an isothermal problem, where the temperature distribution induced Volterra dislocations on the bond. Kuo (1990) and Lee and Park (1995) directly substituted the thermal solution into the modified complex stress and displacement equations (Sadd, 2009). Either way, analytic continuation was used to derive another Hilbert problem. In the previous papers, the fundamental solution of the Hilbert problem had an oscillating singularity near the crack tip, similar to the isothermal case.

The technique for thermal problems has an inherent asymmetry and lack of uniqueness. Through the formulation of a contact problem, Barber (1978) found that if heat flows from a material of high distortivity to one of low distortivity, then the stresses become tensile near the region of contact, which represents a physical contradiction. In the sense of the interface crack problem, with contact zones, Barber and Comninou (1983) resolved this difficulty. The contact zones were assumed to have a subsection in which thermal contact was imperfect, which allows a crack to remain closed instead of interpenetrating.

Just as an applied traction can open a crack, heat flow can also open a crack, which can lead to stress intensity factors and an energy release rate. The singularity associated with thermal stresses, in a single material, was considered by Sih (1962), and was found to be a reciprocal square root. The energy release rate for a multiple-phase material was computed numerically by Xue et al. (2009). In a recent paper (Hasebe and Kato, 2014), a bimaterial strip with an interface crack was considered. The problem was solved by conformally mapping the bimaterial strip to a circle.

Composite materials may have interfaces with many defects, which can be approximately modeled through an assumption of periodically spaced cracks. An isothermal elastic problem with periodically spaced interface cracks was considered by Schmueser and Comninou (1979). The crack was simulated by a continuous distribution of glide and climb dislocations. The superposition of these dislocations, combined with the boundary conditions, resulted in a singular integral equation with a Cauchy kernel. A known infinite sum transformed this integral equation to one with a cotangent kernel. Superposition of these point sources was also used by Block and Keer (2007) for a periodic contact problem.

An alternative technique for solving periodic two-dimensional problems is conformal mapping (Cai and Lu, 2000), which allows use of the analytic function theory technique. The method of Muskhelishvili (1977) is usable for a finite number of cracks in any geometry. If the interface has infinitely many cracks, spaced in periodic intervals, a conformal mapping with a tangent function will map the periodic strip to the entire complex plane, with an additional branch cut (Cai and Lu, 2000). The mapped Riemann-Hilbert problem can then be solved using the method of Muskhelishvili (1977). The solution can be remapped into the original domain and extended periodically. This approach may be more convenient than the superposition technique when the problem is already formulated in terms of complex potentials.

Isothermal interface crack problems are symmetric with respect to reversal of the materials if the boundary conditions are symmetric. However, the thermal loading is not symmetric in most thermoelastic problems. It has been shown that the condition of traction-free cracks applies only if the material of higher temperature is less distortive than the material of lower temperature. If the materials are reversed (or equivalently, if the thermal loading is reversed), the boundary conditions imply that there would be interpenetration instead of crack opening across the full length of the crack (Barber and Comninou, 1983, Martin-Moran et al., 1983, Comninou and Dundurs, 1980). By considering continuous normal tractions and displacements, and zero shear tractions along the crack, the pathological behavior can be eliminated.

In an isothermal problem, Aboudi (1987) modeled imperfect bonds by assuming that the displacement discontinuities are proportional to the traction in the same direction. In a triply periodic system, average strains were found to determine effective elastic parameters. Librescu and Schmidt (2001) considered a problem with anisotropic layers and the same imperfect bonding conditions. Debonding and frictionless contact were respectively considered as limiting cases.

Near the endpoints of a crack, other pathological behavior can result. The research of Leblond and Frelat (2000), Leblond and Frelat (2001) considers a more generally shaped crack or interface crack. In the presence of a mode-II stress intensity factor, the crack tip tends to kink