Mechanics Research Communications 71 (2016) 16–19

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Thermal behavior of elastic columns with second-mode imperfections

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ARTICLE INFO

Article history:
Received 30 August 2015
Received in revised form
29 November 2015
Accepted 30 November 2015
Available online 14 December 2015

Keywords:
Columns
Buckling
Imperfections
Steel
Thermal loading

ABSTRACT

Pin-ended columns having an initial imperfection in a second buckling mode and subjected to thermal loading have been studied in this paper. Based on a nonlinear relationship between strains and displacements, the buckling equilibrium equations are given with the energy method. Then the formulae for the axial compression and transversal displacement are presented. The relationship between the anti-symmetric imperfection and the axial compression has been studied along with the effect of elevated temperature on the initial imperfection. The response of the column in fire to the modified slenderness ratio is investigated. The proposed method has the potential to provide more detailed information for column designs and thus be deployed in future research to minimize the need for expensive laboratory testing.

1. Introduction

The stability of axially-loaded perfect columns is a classical problem. The classical Euler equation is often used to predict the elastic buckling loads (Timoshenko and Gere [1], Simitses [2]). However, the mechanical behavior of steel structures subjected to fire conditions has received increasing attention in recent years. General background about the behavior of steel structures at elevated temperatures can be found in many literatures, such as Buchanan [3] and Wang [4].

The fire resistance of steel columns, which are the main load-carrying members, has been studied analytically, numerically and experimentally by many researchers. A finite difference approach was proposed by Culver [5] to investigate the behavior of wide-flanged steel columns under elevated temperature. Based on the experimental results, Poh and Bennetts [6] proposed a numerical model for the thermal behavior of steel columns. Toh et al. [7] established a simple analytical method to examine the compressive resistance of steel columns at specified temperature levels. A number of steel columns subjected to fire conditions were loaded to their limit states by Yang et al. [8] to check the mechanical behavior of steel columns in fire. Cai and Feng [9] studied the in-plane elastic buckling of a steel column with under thermal loading. Then Cai et al. [10] also investigated the effects of load-dependent supports on the thermal behavior of steel columns. Correia et al. [11] proposed a simplified calculation method for fire design of steel columns with restrained thermal elongation. Alam et al. [12] studied the lateral load resistance of non-insulated steel columns under fire exposure with the finite element method.

It should be noted that the mechanical behavior of steel columns with imperfections is different from perfect columns. Often the shape of the imperfection is assumed to be similar to the first buckling mode (symmetric shape) of the perfect columns [13–15]. However, the imperfection shape of the columns may have the shape of the second buckling mode (anti-symmetric shape). The post-buckling of pinned and cantilevered columns with asymmetric imperfections was studied by Plaut et al. [16]. Rotationally restrained columns having an initial imperfection in an asymmetric mode have been studied by Cai et al. [17]. The investigation of the effects of anti-symmetric imperfections on the thermal behavior of elastic columns has been quite rarely reported in the literatures.

An accurate and reliable analytical formula to predict the behavior of steel columns subject to elevated temperatures is very important, because the cost of the full-scale experiments under fire conditions is high. The objective of this paper is to establish a theoretical model for the mechanical behavior of steel columns under thermal loadings. Moreover, the influence of second-mode imperfections on the thermal behavior of steel columns is also studied. Based on the nonlinear strain–displacement relationship, the principle of virtual work is used to establish the nonlinear equilibrium

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http://dx.doi.org/10.1016/j.mechrescom.2015.11.004
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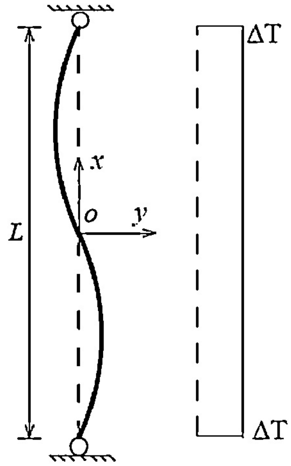

Fig. 1. Columns with anti-symmetric imperfections.

equations. The formulae for the axial force and displacement are also given.

2. Basic theories of columns

A pin-ended column with a second-mode imperfection (anti-symmetric shape) is shown in Fig. 1. The axis system is selected as being centroidal with the origin at mid-height of the column. The relationship between element strains and nodal displacements will be given firstly. An imperfect column with a small initial curvature is considered in this paper. All the assumptions for normal slender columns are satisfied except that the column now has an initial shape.

The initial curvature based on the second buckling mode of steel column can be described by

$$y = a\sin\left(\frac{2\pi x}{L}\right) \tag{1}$$

where L is the span of the column.

Then the nonlinear strain–displacement relationship for any point on the cross section of steel columns can be written as

$$\varepsilon = \varepsilon_m + \varepsilon_b = \varepsilon_m + yk \tag{2}$$

where ε_m is the membrane strain, ε_b is the bending strain, and κ is the change in the curvature.

The displacements of any point on the cross section in the ox and oy directions are denoted as $u(x)$ and $v(x)$, respectively. If the undeformed length of the column is ds and deformed length of the column is ds^*, then with the assumption of small strains, the membrane strain of steel columns is given by

$$\varepsilon_m = \frac{1}{2}\frac{(ds^*)^2 - (ds)^2}{(ds)^2} \tag{3}$$

where $(ds)^2 = (dx)^2 + (dy)^2$, $(ds^*)^2 = (dx + du)^2 + (dy + dv)^2$.

The imperfect columns are assumed that $(dy/dx)^2 \ll 1$ and $du/dx \ll 1$, so that Eq. (3) can be expressed as

$$\varepsilon_m = u' + g(x)v' + \frac{1}{2}(v')^2 \tag{4}$$

in which $()' = d()/dx$, $g(x) = \frac{2a\pi}{L}\cos\left(\frac{2\pi}{L}x\right)$.

For the steel column with a small initial curvature, the bending strain can be written as

$$\varepsilon_b = -yv''. \tag{5}$$

In addition, the strain produced by the evaluated temperature ΔT can be given as

$$\varepsilon_t = \alpha\Delta T \tag{6}$$

where ε_t is the thermal strain, ΔT is the temperature increment relative to its ambient value, and α is the coefficient of thermal expansion that is set to $1.2 \times 10^{-5}/^\circ$C in this study.

Therefore, the total strain of the column can be obtained as

$$\varepsilon = \varepsilon_e + \varepsilon_t \tag{7}$$

where ε_e denotes the mechanical elastic strain, which consists of two components, the axial strain ε_{em} and the bending strain ε_{eb}. All strains are defined as positive in tension. From Eqs. (2)–(7), the mechanical axial strain and bending strain of steel column can be given as

$$\varepsilon_{em} = u' + g(x)v' + \frac{1}{2}(v')^2 - \alpha\Delta T \quad \varepsilon_{eb} = -yv'' \tag{8}$$

The differential equations of equilibrium for a pin-ended column with imperfections under thermal loadings can be obtained with the principle of virtual work which requires

$$\int_{-L/2}^{L/2}[(EA)_{eq}(\delta u' + g(x)\delta v' + v'\delta v')\varepsilon_{em} + (EI)_{eq}v''\delta v'']dx = 0 \tag{9}$$

for all sets of kinematically admissible virtual displacements δu and δv. In Eq. (9), temperature-dependent cross-section properties $(EA)_{eq}$ and $(EI_z)_{eq}$ are defined as

$$(EA)_{eq} = \int_A E(y)dA, \quad (EI_z)_{eq} = \int_A E(y)y^2dA$$

where $E(y)$ is the temperature-dependent Young's modulus at the coordinate y.

Integrating Eq. (9) by parts leads to

$$\begin{aligned}
&(EA)_{eq}\varepsilon_{em}\delta u|_{-L/2}^{L/2} - \int_{-L/2}^{L/2}(EA)_{eq}\varepsilon'_{em}\delta u dx + (EA)_{eq}g(x)\varepsilon_{em}\delta v|_{-L/2}^{L/2} \\
&- \int_{-L/2}^{L/2}(EA)_{eq}((g(x)\varepsilon_m)'\delta v dx + (EA)_{eq}v'\varepsilon_m\delta v|_{-L/2}^{L/2} \\
&- \int_{-L/2}^{L/2}(EA)_{eq}(v'\varepsilon_m)'\delta v dx + (EI)_{eq}v''\delta v'|_{-L/2}^{L/2} - (EI)_{eq}v'''\delta v|_{-L/2}^{L/2} \\
&+ \int_{-L/2}^{L/2}(EI)_{eq}v^{iv}\delta v dx v = 0
\end{aligned} \tag{10}$$

in which $v^{iv} = d^4v/dx^4$.

Then the differential equilibrium equations of imperfect columns in the axial direction can be derived from Eq. (10) as

$$-(EA)_{eq}\varepsilon'_{em} = 0 \tag{11}$$

From Eq. (11), it can be found that the membrane strain ε_{em} is constant. It can be written as

$$\varepsilon_{em} = -\frac{N}{(EA)_{eq}} \tag{12}$$

where N is the actual axial compression in the steel column.

In addition, the differential equilibrium equation of imperfect columns in the transversal direction can be derived from Eqs. (10) and (12) as

$$(EI)_{eq}v^{iv} + Nv'' + Ng'(x) = 0 \tag{13}$$

For simplicity, the following new parameter is introduced

$$\mu^2 = \frac{N}{(EI)_{eq}}, \tag{14}$$