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Influence of curved struts, anisotropic pores and strut cavities on the effective elastic properties of open-cell foams

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ABSTRACT

This work focuses on the modelling of the local foam geometry (strut and node) and analyses its influence on the effective elastic properties. A periodic KELVIN cell is used as a simplified open cell structure. This allows detailed studies of often neglected local geometric details like strut thickness variations, strut curvature, node thickness, node and strut cavities. The foam geometry is described using implicit functions in BLINN transformations, which is a versatile modelling method leading to detailed finite element models. The finite element method together with a computational homogenisation technique is used to determine the effective elastic properties. These are presented for KELVIN cells having a constant porosity, but varying strut curvature and cross-section shape. Geometrical anisotropy and cavities in the struts and nodes are considered as well.

1. Introduction

Foam structures are widely used in technical applications as filters, lightweight structures, energy absorbers and others. Beside the bulk material properties the local geometry determines the effective physical properties of foam based structures. In the special case of open cell reactive filters (Aneziris et al., 2013; Emmel and Aneziris, 2012), mechanical, thermal and chemical properties are of great interest. In this paper we will concentrate on the effective elastic properties of ceramic open cell foams. Such structures are stochastic spatial networks of struts that are rigidly connected at common nodes. One typical production route is to coat a polyurethane (PU) foam with a liquid ceramic slurry and to sinter it (Schwartzwalder, 1963). This leads to a structure, which has the same topological properties as the PU foam. Only the strut cross sections change and especially at the nodes material concentrations occur. During the sinter process the PU pyrolyses and cavities remain inside the struts and nodes.

Real metallic and ceramic foams based on PU foams often contain a systematic pore anisotropy. Hence, the influence on the mechanics of the foam structure is of big interest. A first study using TIMOSHENKO beam models was performed by Gong et al. (2005). There, a PLATEAU-border shaped cross section for the struts was applied. The size of the cross section was non-uniform along the strut axis to fit a computer tomography result of a PU foam.

Experimental studies of such structures are cost and time intensive, whereas numerical simulations by finite element (FE) models are easily feasible. A versatile method to model foam structures with certain geometrical details is the model generation based on implicit functions in BLINN transformation (Storm et al., 2013). To define the shape of the struts and nodes only a small number of parameters is needed, which makes systematic studies possible. There are other modelling methods for foams, e.g. based on minimal surfaces (Buffel et al., 2014), local adaptive morphology (Lautensack et al., 2008) and non uniform rational B-splines (Michailidis et al., 2014). Furthermore, there are several ways of reconstructing models from computer tomography images.

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Typical foam structures comprising thousands of stochastic cells having a detailed finite element mesh require a large number ($\gg 10^6$) of degrees of freedom. To decrease the model size we use a single periodic KELVIN cell (Thomson, 1887) as basic structure, which is a reasonable choice, since the relevant deformation processes (strut bending, stretching, shearing and torsion) are similar to real (stochastic) foams considered in this project. But it is well known that deterministic structures and other approximated foam models (e.g. from VORONOI, LAGUERRE tessellation or Surface Evolver) differ in some properties like mean strut length compared to real foams (Kraynik, 2006; Kraynik, 2003).

The geometry variations, introduced in this paper lead to theoretical foam models which allow the analysis of the corresponding changes of the effective elastic behaviour. The effect of some basic properties like strut thickness, node thickness and rounding on the elastic response of KELVIN foams has been already published (Storm et al., 2013). Other properties like the slight strut curvature, the geometrical anisotropy and the cavity influence, which are results of the production route are not well known and therefore within the focus of this study.

2. Model description

2.1. Volumetric model

A method based on implicit functions (Storm et al., 2013) is used to generate the volumetric foam models. The models are derived from the deterministic foam structure of the KELVIN cell to analyse the influence of strut curvature, pore anisotropy and strut cavities. The structure of the characteristic KELVIN unit cell is shown in Fig. 1. All struts have a length of l_0, thus the cell model length is $l = 2\sqrt{2}l_0$. To model a KELVIN cell and its geometrical variations the following base structures for struts and cavities are used:

- hyperboloid with rounded ends (Fig. 2a)
- curved hyperboloid with rounded ends (Fig. 2b)
- parabolic, equilateral three-sided prism (Fig. 2c)
- curved parabolic, equilateral three-sided prism (Fig. 2d).

The parameters for changing the local foam geometry are the middle radius r_m, the end (sphere) radius r_e, the triangle hight h_t, the strut curvature radius r_c and a function for the surface curvature $f(x)$. All the base elements are combined into one single implicit function (Storm et al., 2013), applying the BLINN transformation, which allows to smooth the transition between neighbouring elements by the BLINN parameter a. From the final implicit function of the foam an isosurface is extracted, the resulting surface mesh is simplified and a volumetric tetrahedron mesh is constructed.

An anisotropic characteristic KELVIN cell is created by stretching with a factor β for a single coordinate of all point coordinates (see Fig. 3). Subsequently, the volumetric model is generated by the previously mentioned way.

The method to create struts with a cavity is discussed in Storm et al. (2013). Here, a combination of the hyperboloid basis element with rounded ends and the parabolic, equilateral three-sided prism is used. The cross-section of the prism is assembled by three parabolas (Fig. 2e). The function for a single parabola is:

$$f(x) = \frac{2}{9r_t}x^2 + \frac{r_t}{3}, \tag{1}$$

where r_t denotes the radius of the circumcircle. Eq. (1) represents a parabolic fit of the characteristic hypocycloidal shape of a soap foam, which was studied e.g. by Thomson (1887) and Gong et al. (2005). This is a good approximation of the ideal shape of a soap froth known as PLATEAU borders (Plateau, 1873). For uncurved struts the angles between two adjacent struts at a foam node are not equal. If the strut axes coincide with the centroid of the cross sections, the edges of the equilateral, triangular basis elements do not meet exactly (see Fig. 4a), which was also noticed and illustrated by Gong et al. (2005). This can be corrected by shifting the strut axis about

$$d = r_t\left(\frac{1}{2} - \frac{\sqrt{3}}{4}\right) \tag{2}$$

away from the centre of the adjacent four-sided facet (see Fig. 4).

Curved struts (Thomson, 1887) are realised with implicit functions by a transformation into a polar coordinate system for the base elements (Fig. 2b). The transformation in Fig. 5 is defined for $r_c \in (0, 0.5l_0]$:

$$x'_1 = \sqrt{(x_1 + b)^2 + x_2^2} - r_c \tag{3}$$

$$x'_2 = \frac{r_c}{s}\arccos\left(\frac{x_1 + b}{\sqrt{(x_1 + b)^2 + x_2^2}}\right) \tag{4}$$

$$x'_3 = x_3 \tag{5}$$

with

$$b = r_c\sqrt{1 - \left(\frac{l_0}{2r_c}\right)^2} \tag{6}$$

$$s = \frac{2r_c}{l_0}\arcsin\left(\frac{l_0}{2r_c}\right) \tag{7}$$

If the curvature radius is set to

$$r_c = \frac{l_0\sqrt{3}}{2 - \sqrt{2}}, \tag{8}$$

then the angle between neighbouring struts becomes $\alpha = \arccos\left(-\frac{1}{3}\right)$, which corresponds to the tetrahedral angle predicted by PLATEAUS's law.

2.2. Homogenization

Using the periodic KELVIN cell as a representative volume element (RVE) the effective properties have to be determined. The relations between the stress and strain fields