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A directional modification of the Levkovitch–Svendsen cross-hardening model based on the stress deviator

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ABSTRACT

In the original Levkovitch–Svendsen cross-hardening model parallel and orthogonal projections required for the yield surface evolution with respective dynamic and latent hardening effects are associated with the unit plastic flow direction $\boldsymbol{n}_p = \dot{\mathbf{E}}_p/|\dot{\mathbf{E}}_p|$. This work gives a detailed investigation regarding the consequences and proposes the use of the so–called radial direction $\boldsymbol{n}_s = [\mathbf{S} - \mathbf{X}]/|\mathbf{S} - \mathbf{X}|$ instead where $\mathbf{S} = \mathrm{dev}(\boldsymbol{\sigma})$. It is shown that for an initially plastically anisotropic material under load paths with proportional stresses the original model brings a continuous directional change in the plastic strains. Eventually, even if the dynamic hardening component is bypassed, the material model predicts additional strengthening in loading direction due to latent hardening. In this undesired response, the broken coaxiality of the stress deviator and plastic strain rate tensor with initial anisotropy is the cause. This entanglement of isotropic/kinematic hardening and latent hardening creates difficulties – especially in the parameter identification even for the simplest uniaxial loading. The introduced modification to the model remedies this undesired feature and, hence, makes it possible to isolate the hardening sources during parameter identification stage. The discussions are supported by analytically and numerically derived yield loci for various scenarios. Our analytical studies allow definition of critical material parameter limits for the latent hardening parameter s_l in terms of the initial anisotropy and the constant stress deviator ratio.

1. Introduction

The macroscopic material behavior of crystalline solids is strongly related to a present or emergent underlying microstructure. The microstructure itself is a result of different physical mechanisms at lower length scales. As the level of deformation gets large enough, microstructural defects are permanently reconfigured to some characteristic dislocation patterns, i.e., labyrinth-type dislocation structure (Jin and Winter, 1984) or dislocation cell structures (Tabata et al., 1983), to name but a few. As we are restricting our attention in this work to phenomenological plasticity models which are applicable to forming processes, the evolution of the microstructure is accounted for by an evolving anisotropy tensor of the material (Feigenbaum and Dafalias, 2007; Wang et al., 2008; Pietryga et al., 2012; Shi and Mosler, 2013; Barthel et al., 2013; Behrouzi et al., 2014). One aspect of this anisotropy is the appearance of cross-hardening effects for orthogonal loading-path changes (Bouvier et al., 2006).

Let $\Phi = \sqrt{[\boldsymbol{\sigma} - \boldsymbol{X}]^{\top}\mathbb{B}[\boldsymbol{\sigma} - \boldsymbol{X}]} - h(\gamma) \leqslant 0$ denote the flow potential with γ representing the equivalent plastic strain and $h(\gamma)$ the associated flow stress. $\mathbb{B} = \mathbb{A} + \mathbb{H}$ is the structural tensor where the additive component $\mathbb{A}$ encapsulates

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material's inherent anisotropy, whereas $\mathbb{H}$, the anisotropy induced by plastic flow with the initial value $\mathbb{H}_0 = \mathbb{O}$ for the loading time $t = 0$. Hence, along with these definitions, we assume $\dot{\mathbb{A}} = \mathbb{O}$ and $\dot{\mathbb{H}} \neq \mathbb{O}$ where $\mathbb{O}$ represents the fourth-order zero tensor. In associated plasticity theory, the direction of plastic flow coincides with the normal to the yield locus at the point of loading as defined by $\boldsymbol{n}_p = \dot{\mathbf{E}}_p/|\dot{\mathbf{E}}_p| = \partial_{\boldsymbol{\sigma}}\Phi/|\partial_{\boldsymbol{\sigma}}\Phi| = \mathbb{B}[\boldsymbol{\sigma} - \boldsymbol{X}]/|\mathbb{B}[\boldsymbol{\sigma} - \boldsymbol{X}]|$, where $\dot{\mathbf{E}}_p$ denotes the plastic strain rate tensor. Additionally, one may define a so–called radial direction $\boldsymbol{n}_s = [\mathbf{S} - \boldsymbol{X}]/|\mathbf{S} - \boldsymbol{X}|$ as a normalized difference of the stress deviator $\mathbf{S} = \mathrm{dev}(\boldsymbol{\sigma})$ and the back stress tensor $\boldsymbol{X}$. In Levkovitch–Svendsen cross-hardening model (Levkovitch and Svendsen, 2007) parallel and orthogonal projections used in definition of $\dot{\mathbb{H}}$ with respective dynamic and latent hardening effects require a certain direction. In retrospect, the original model formulation of Levkovitch–Svendsen (Levkovitch and Svendsen, 2007) as well as the subsequent model formulations and extensions (Barthel et al., 2008; van den Boogaard and van Riel, 2009; Clausmeyer et al., 2011; Shi and Mosler, 2013; Barthel et al., 2013; Behrouzi et al., 2014), this direction is selected as the plastic flow direction $\boldsymbol{n}_p$. One exception in this context is the work by Pietryga et al. (2012), where the evolution of the original anisotropic flow tensor is formulated based on the radial direction $\boldsymbol{n}_s$. However, the authors name this direction as the plastic flow direction without pointing out its difference from the previous model formulations as well as the consequences of such a usage.

In this work, restricting our analysis to the Levkovitch–Svendsen model, we investigate the consequences of the use of these two distinct directions in the definition of the evolution of the fourth-order structural tensor $\mathbb{H}$ for materials with initial plastic anisotropy. Our analysis reveals a drawback, hereby named as the hardening entanglement, in the original model formulation based on the direction of plastic flow $\boldsymbol{n}_p$. For an initially plastically anisotropic material under loading paths with proportional stresses, the original model brings a continuous directional change in the rate of plastic strain tensor, hence, in the associated normal $\boldsymbol{n}_p$. Eventually, even if the dynamic hardening component is bypassed, the material model predicts additional hardening in loading direction. Under uniaxial stresses, as a consequence, a continuous change in the Lankford's coefficient r_0 is observed. Here, the broken coaxiality in between $\boldsymbol{n}_p$ and $\boldsymbol{n}_s$ is the cause. Note that only for the case of von Mises isotropic plasticity one has $\boldsymbol{n}_p \equiv \boldsymbol{n}_s$. This entanglement of isotropic/kinematic hardening and latent hardening creates difficulties – especially in the parameter identification even for the simplest uniaxial loading by precluding the isolation of the hardening sources. We show that the proposed modification by using the radial direction $\boldsymbol{n}_s$ remedies these undesired features and support this statement by analytical as well as numerical examples.

2. Generalization of Levkovitch–Svendsen evolving yield locus

Assuming that the initial orthotropy axes are aligned with the loading directions, the principal axes of deformation do not rotate and in order to simplify the context the influence of the backstress is neglected. Thus, the Levkovitch–Svendsen yield locus reads

$$\Phi = \sqrt{\boldsymbol{\sigma}^{\top}\mathbb{B}\boldsymbol{\sigma}} - h(\gamma) \leq 0. \tag{1}$$

In absence of shear components, $\boldsymbol{\sigma}$ is regarded as a 3×1 vector with $\boldsymbol{\sigma} = (\sigma_{11}, \sigma_{22}, \sigma_{33})^{\top}$ aligned with principal axes. Here, a simple isotropic hardening function is assumed of the form

$$h(\gamma) = \sigma_\infty - [\sigma_\infty - \sigma_{y0}]\exp(-m\gamma), \tag{2}$$

where σ_{y0} and σ_∞ denote the initial and saturated yield stresses, respectively, and m represents the saturation rate. Moreover, $\mathbb{B} = \mathbb{A} + \mathbb{H}$. Here both $\mathbb{A}$ and $\mathbb{H}$ represent 3×3 matrices with the former being the Hill'48–type structural matrix. For $\dot{\mathbb{H}}$ we propose the following generalization

$$\dot{\mathbb{H}} = \dot{\gamma}c_d\left[s_d\mathbb{N}_g - \mathbb{H}_d\right] + \dot{\gamma}c_l\left\{s_l\left[\mathbb{I}_{\mathrm{dev}} - \mathbb{N}_g\right] - [\mathbb{H} - \mathbb{H}_d]\right\}, \tag{3}$$

where $\mathbb{N}_g := \boldsymbol{n}_g \otimes \boldsymbol{n}_g$ with $\boldsymbol{n}_g$ defining the unit tensor used for parallel and orthogonal projections. We use the term generalization, since unlike Levkovitch –Svendsen original model which fixes $\boldsymbol{n}_g$ as the unit direction of plastic flow with $\boldsymbol{n}_g \equiv \boldsymbol{n}_p$, we do not define a specific choice for $\boldsymbol{n}_g$. $\mathbb{I}_{\mathrm{dev}}$ is the matrix representation of the fourth-order deviatoric projection (idempotent), that is $\mathbb{I}^n_{\mathrm{dev}} = \mathbb{I}_{\mathrm{dev}}$ for $n \geqslant 1$ with

$$\mathbb{I}_{\mathrm{dev}} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \tag{4}$$

and $\mathbb{H}_d$ is defined by

$$\mathbb{H}_d = \left[\boldsymbol{n}_g^{\top}\mathbb{H}\boldsymbol{n}_g\right]\boldsymbol{n}_g \otimes \boldsymbol{n}_g. \tag{5}$$

An immediate integration of Eq. (3) is possible for constant $\boldsymbol{n}_g$, thus constant $\mathbb{N}_g$, no matter which direction it represents, giving an additive form $\mathbb{H} = \mathbb{H}_d + \mathbb{H}_l$ with[1]

$$\mathbb{H}_d = s_d f_{c_d}(\gamma)\mathbb{N}_g \quad \text{and} \quad \mathbb{H}_l = s_l f_{c_l}(\gamma)\left[\mathbb{I}_{\mathrm{dev}} - \mathbb{N}_g\right], \tag{6}$$

where we used the abbreviations

$$f_{c_l}(\gamma) = 1 - \exp(-c_l\gamma) \quad \text{and} \quad f_{c_d}(\gamma) = 1 - \exp(-c_d\gamma). \tag{7}$$

Here, c_d and s_d denote the saturation rate and magnitude associated with the dynamic part $\mathbb{H}_d$, where c_l and s_l are the saturation rate and magnitude associated with the latent part $\mathbb{H}_l := \mathbb{H} - \mathbb{H}_d$, respectively.[2] $\mathbb{H}_d$ accounts for the strength of the dislocation structures linked to the slip systems which are currently active whereas the strength of the dislocation structures of currently inactive slip systems is encapsulated in $\mathbb{H}_l$. Hence, for $s_d = 0$ until a load path change occurs one expects that the material response will be dictated purely by isotropic hardening. For $\gamma = 0$ one has $\mathbb{H}_0 = \mathbb{O}$ as required.

[1] In the presence of kinematic hardening evolving in direction of plastic flow $\boldsymbol{n}_p, \boldsymbol{n}_s$ is not constant in general.

[2] Both $\mathbb{H}_d$ and $\mathbb{H}_l$ have distortional effects which cannot be represented by isotropic or kinematic hardening.