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A refined theory with stretching effect for the dynamics analysis of advanced composites on elastic foundation

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ABSTRACT

This paper presents a free vibration analysis of functionally graded plates (FGPs) resting on elastic foundation by using a higher order shear deformation theory (HSDT) in which the stretching effect is included. The highlight of this theory is that, in addition to including the thickness stretching effect ($\varepsilon_{zz} \neq 0$), the displacement field is modeled with only 4 unknowns, which is even less than the first order shear deformation theory (FSDT). The elastic foundation follows the Pasternak (two-parameter) mathematical model. The governing equations are obtained through the Hamilton's principle. These equations are then solved via Navier-type, closed form solutions. The fundamental frequencies are found by solving the eigenvalue problem. The degree of precision of the current solutions can be noticed by comparing it with the 3D and other closed form solutions available in the literature.

1. Introduction

Functionally graded materials (FGMs) are a type of heterogeneous composite material in which the properties change gradually over one or more directions. This material is produced by mixing two or more materials in a certain volume ratio (for example, metal and ceramic). Classical composites structures such as fiber reinforced plastics (FRPs) suffer from discontinuity of material properties at the interface of the layers and constituents. These problems can be decreased by gradually changing the volume fraction of constituent materials and tailoring the material for the desired application.

On the other hand, the elastic foundations have wide applications in engineering. Winkler (1867) presented a one-parameter model to describe the mechanical behavior of elastic foundations, whereas Pasternak (1954) presented a two-parameter model, additionally, this model considers the shear deformation between the springs. Consequently, Winkler model can be considered as a special case of Pasternak model by setting the shear modulus to zero.

Starting this decade, Hosseini-Hashemi et al. (2010) presented an analytical solution for free vibration analysis of moderately thick rectangular plates, which are composed of FGMs and supported by either Winkler or Pasternak elastic foundations. Hasani Baferani et al. (2011) presented a vibration analysis of FGPs resting on two parameter elastic foundation using Reddýs HSDT.

Thai and Choi (2012) presented a HSDT for free vibration of functionally graded plates on elastic foundation. The authors present a displacement field based on the assumptions that the in-plane and transverse displacements consist of bending and shear components. Sheikholeslami and Saidi (2013) studied the free vibration analysis of FGPs resting on two-parameter elastic foundation using a HSDT and an analytical approach. The authors expanded the displacement components in the thickness direction using the Legendre polynomials. Zenkour et al. (2013) investigated the bending response of an orthotropic

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rectangular plate resting on two-parameter elastic foundations under thermo-mechanical loadings. Akavci (2014) presented a free vibration analysis of FGPs on elastic foundation applying a non-polynomial HSDT and an optimization procedure. Al Khateeb and Zenkour (2014) presented the bending analysis of FGPs resting on elastic foundations. The authors considered the influence of temperature and moisture by using a refined HSDT.

Commonly, non-polynomial share strain shape functions, such as trigonometric, trigonometric hyperbolic, exponential, etc., can be used in the mathematical formulation of HSDTs. However, the thickness expansion modeling, $g(z)$, is conditioned by the in-plane displacement model, $f(z)$, i.e. the transverse shear strain function is an even function which is the derivative of the in-plane shear strain shape function ($g(z) = f'(z)$). Therefore, there is no freedom to choose the shear strain shape function of the thickness displacement field. The present formulation has that freedom, and infinite hybrid type shear deformation theories (polynomial or non-polynomial or hybrid type) can be created, see for example Mantari and Guedes Soares (2012, 2014a,b).

In the present paper, a generalized formulation with stretching effect for the vibrational analysis of FGPs on elastic foundation is presented. The highlight of this theory is that, in addition to including the thickness stretching effect ($\varepsilon_{ZZ} \neq 0$), the displacement field is modeled with only 4 unknowns, which is even less than the FSDT and do not need shear correction factor. The mechanical properties of the plates are assumed to vary in the thickness direction according to a power law distribution in terms of the volume fractions of the constituents. The governing equations of FGPs resting on elastic foundation are derived by employing the Hamilton's principle. These motion equations are then solved via Navier solution. As a result, fundamental frequencies are found by solving eigenvalue problem. The accuracy of the present code is verified by comparing it with HSDT's solutions available in literature.

The paper is organized as following. Section 2 outlines the mathematical modeling of the HSDT. Theoretical formulation of FGMs, displacement field, kinematic and constitutive relations, Hamilton's Principle, and the governing equations. Section 3 describes the solution methodology. Section 4 is about results and discussions. Finally, further general aspects are given in conclusions.

2. Theoretical formulation

2.1. Functionally graded plates

A rectangular plate of uniform thickness "h", length "a", and the width "b", made of a FGM and resting on elastic foundation is shown in Fig. 1. The rectangular Cartesian coordinate system x, y, z, has the plane $z = 0$, coinciding with the mid-surface of the plate. The material properties vary through the thickness with a power law distribution, which is given below (see Fig. 2):

$$P_{(z)} = (P_t - P_b)V_{(z)} + P_b$$
$$V_{(z)} = \left(\frac{z}{h} + \frac{1}{2}\right)^p$$
$$-\frac{h}{2} \leqslant z \leqslant \frac{h}{2} \tag{1a-c}$$

where P denotes the effective material property, P_t and P_b denote the property of the top and bottom faces of the plate, respectively, and "p" is the exponent that specifies the material variation profile through the thickness. The effective material properties of the plate, including Young's modulus, E, and shear modulus, G, vary according to Eqs. (1a,b), and Poisson ratio, "v" is assumed to be constant.

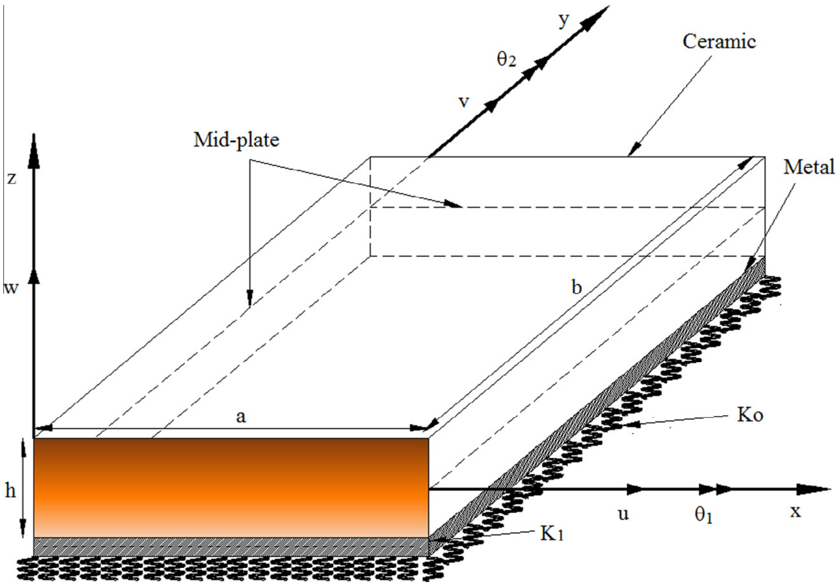

Fig. 1. Geometry of a functionally graded plate resting on two parameter elastic foundation.