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Modeling the effect of DC on the creep of metals in terms of the synthetic theory of irrecoverable deformation

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ABSTRACT

The objective of this paper is to model the effect of direct current upon the steady-state creep of metals. The modeling of this phenomenon has been accomplished in terms of the synthetic theory of permanent deformation. The constitutive equation of the theory is supplemented by a term taking into account the passing of DC. Results obtained in terms of this theory show good agreement with experimental data.

1. Introduction

In this paper, we will discuss the steady-state creep rate of metals subjected to the passage of electric currents of high current densities. Researchers studying this phenomenon report an increase in their steady-state creep rate due to the passage of DC (Braunovic et al., 2006; Zhao and Yang, 2014; Kinney et al., 2009; Zhao et al., 2012; Yang and Zhao, 2010; Sanmartin et al., 1983; Wang-Yun et al., 2015), see Figs. 2 and 3.

The DC effect is suggested to be caused by the following:

(i) DC-induced Joule heating causing a change in local temperature and resulting in time-dependent plastic deformation (Chen and Yang, 2008; Chiao and Lin, 2000; Yang, 2009).
(ii) The momentum exchange between moving electrons and lattice atoms reduces the energy barrier and increases the migration velocity of atoms (Chen and Yang, 2008, 2010, 2011).
(iii) The intensification of the current field assisted sliding rate and diffusional creep (Kumar and Dutta, 2011; Shao et al., 2012).

According to Braunovic et al. (2006), Zhao and Yang (2014) and Kinney et al. (2009), among the phenomena accompanying the passage of current listed above, the two latter are dominant.

The influence of current upon irrecoverable deformation has been registered for such metals and their alloys as copper, nickel, aluminum, tin, etc. (Braunovic et al., 2006; Zhao and Yang, 2014; Kinney et al., 2009; Sanmartin et al., 1983). Researches dealing with the DC effect upon creep are mostly of experimental fashion. Relating the modeling of this phenomenon, we can invoke at least pure phenomenological result that the creep rate is a linear function of current intensity (Zhao and Yang, 2014).

This work is focused on modeling the creep behavior of polycrystalline metals under the simultaneous action of electrical current at a given temperature. The mathematical apparatus we use is the synthetic theory of irrecoverable deformation (Rusinko and Rusinko, 2009, 2011).

The synthetic theory prove oneself to be an effective and far reaching approach to model various non-classical problems of irrecoverable deformation such as the influence of ultrasound upon plastic/creep deformation (Rusinko, 2011, 2014), negative creep observed at step-wise loading (Rusinko, 2012), the effect of preliminary mechanical thermal treatment upon the steady-state creep (Rusinko, 2012), etc. The main advantage of the theory consist in

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(a) macrodeformation of body is calculated as the sum of microdeformations thereby accounting for the processes developing on its microlevel and (b) microdeformation is directly related to the main carries of deformation, defects of the crystalline structure of material.

To test the synthetic theory we construct plots for the steady-state creep of tin, whose plastic deformation (Yang and Li, 2007; Hamada et al., 2010) has already been modeled in Rusinko (2015). This metal was singled out of others because it is employed in many ways: as solder for joining pipes or electric circuits (Abtew and Selvaduray, 2000), pewter, bell metal, babbit metal and dental amalgams. The niobium–tin alloy is used for superconducting magnets, tin oxide is used for ceramics and in gas sensors (as it absorbs a gas its electrical conductivity increases and this can be monitored).

2. Synthetic theory

The synthetic theory is based on the Batdorf–Budiansky slip concept (Batdorf and Budiansky, 1949) and the Sanders flow theory (Sanders, 1954) and deals with small irrecoverable (plastic/creep) deformations of hardening materials. Its detailed description can be found in (Rusinko and Rusinko, 2009, 2011); here, we outline only central tenets and equations of the synthetic theory needed for further calculations.

The modeling of irrecoverable deformation takes place in the three-dimensional subspace ($\mathcal{R}^3$) of the Ilyushin five-dimensional space of stress deviators, $\mathcal{R}^5$, (Ilyushin, 1963). The loading process is expressed by a stress vector, $\tilde{\mathbf{S}}$, whose components are converted from the stress deviator tensor components – S_{ij} $(i, j = x, y, z)$ – as follows (Rusinko and Rusinko, 2009, 2011):

$$\tilde{\mathbf{S}}\left[\sqrt{3/2}S_{xx}, S_{xx}/\sqrt{2}+\sqrt{2}S_{yy}, \sqrt{2}S_{xz}\right] \in \mathcal{R}^3 \tag{1}$$

Permanent deformation at a point of body is expressed via strain vector which is defined as

$$\tilde{\mathbf{e}} = \int_\alpha\int_\beta\int_\lambda \varphi_N \tilde{\mathbf{N}} dV, \quad dV = \cos\beta\, d\alpha\, d\beta\, d\lambda. \tag{2}$$

Its components are converted to the strain-deviator tensor components, e_{ij} $(i, j = x, y, z)$, as

$$e_1 = \sqrt{3/2}e_{xx}, \quad e_2 = e_{xx}/\sqrt{2}+\sqrt{2}e_{yy}, \quad e_3 = \sqrt{2}e_{xz}. \tag{3}$$

In Eq. (2), φ_N is a strain intensity, which is an average measure of plastic slip developing within a microvolume (slip system, $\mathbb{V}_0$). Eq. (2) states that plastic/creep deformation at a point of body (macrodeformation) is a sum of slips within microvolumes (microdeformation) composing the point of body which is assumed to be an elementary volume ($\mathbb{V}$), $\mathbb{V} = \Sigma\mathbb{V}_0$.

Slip systems are presented/modeled by tangent planes drawn through every point of yield surface in $\mathcal{R}^5$. Since the analysis of five-dimensional figures is extremely complicated, we work with the projection of the five-dimensional yield surface on $\mathcal{R}^3$, which is a sphere of radius $\sqrt{2/3}\sigma_P$ and corresponds to the von-Mises yield criterion,

$$S_1^2+S_2^2+S_3^2 = 2/3\sigma_P^2, \tag{4}$$

where σ_P is a creep limit of material in uniaxial tension. The location of a plane in $\mathcal{R}^3$ is established via unit normal vector $\tilde{\mathbf{N}}(\alpha, \beta, \lambda)$ and the distance to the plane, H_N.

According to Sanders (1954), in a virgin state, the sphere (4) can be thought of the inner envelope of equidistant tangent planes. To establish a *hardening rule*, Sanders extended the provision that a surface can be constructed as an inner envelope of planes to the case of loading as well. In the course of loading, the vector $\tilde{\mathbf{S}}$ displaces on its endpoint a set of planes from their initial position, i.e. from sphere (4). The displacement of plane on the endpoint of stress vector symbolizes the development of plastic microdeformation within the slip system.

The distance to a plane characterizes the degree of the hardening of material. Indeed, the greater the H_N is, the greater the stress vector will be needed to reach the plane, i.e. to induce plastic shift within the corresponding slip system. The condition that a plane is located at the endpoint of stress vector is

$$H_N = \tilde{\mathbf{S}}\cdot\tilde{\mathbf{N}} = S_1N_1+S_2N_2+S_3N_3. \tag{5}$$

To reflect the well-known fact that a plastic flow of material is accompanied by the nucleation, multiplication, movement and interaction of the irregularities of crystalline grid (dislocations, point defects, twins, etc.), we introduce defect intensity: an average measure of the defects (ψ_N) formed during plastic deforming within $\mathbb{V}_0$

$$H_N^2 = \psi_N + 2/3\sigma_P^2. \tag{6}$$

The strain intensity within one slip system (φ_N) is related to the defects intensity (ψ_N) and time (t) by the following differential equation (Rusinko and Rusinko, 2009; Rusinko and Rusinko, 2011):

$$d\psi_N = r d\varphi_N - K\psi_N dt, \quad r = const, \quad K = K(\Theta, \sigma_{eff}). \tag{7}$$

This formula tells us that the number of defects is governed by two processes: it grows due to an increase in irrecoverable straining ($d\varphi_N$) and simultaneously decreases (undergoes relaxation) in the course of deforming ($-K\psi_N dt$). Depending on loading- and temperature regime one of the term in (7) can dominate over another, or both of them may manifest itself.

For the case of steady-state creep, when $S = const$, Eqs. (5)–(7) give that

$$\dot{\varphi}_N = \frac{K}{r}\psi_N = \frac{K}{r}\left(\left(\tilde{\mathbf{S}}\cdot\tilde{\mathbf{N}}\right)^2 - 2/3\sigma_P^2\right) = const, \tag{8}$$

$$K = K_1(\Theta)K_2\left(\sigma_{eff}\right), \quad K_1(\Theta) = \exp\left(-\frac{Q}{R\Theta T_m}\right),$$
$$K_2\left(\sigma_{eff}\right) = \frac{9\sqrt{3}cr}{2\sqrt{2}\pi}\sigma_{eff}^{k-2}, \tag{9}$$

where r, c and k are model constants; Θ and $\sigma_{\rm eff}$ are the homology temperature and effective stress, respectively. As we can see the function K_2 is consonant with the Bailey–Norton law (power law creep) and, thus, follows its logic about the effect of power index (in our case it is k) on the creep deformation.

For the case of uniaxial tension ($S_1 = \sqrt{2/3}\sigma_x$, $N_1 = \cos\alpha\cos\beta\cos\lambda$; Rusinko and Rusinko, 2011; Rusinko, 2011) Eq. (2) gets

$$\dot{e}_1 = \frac{2K}{3r}\int_{-\alpha_1}^{\alpha_1}\int_{-\beta_1}^{\beta_1}\int_0^{\lambda_1}\left[\left(\sigma_x\cos\alpha\cos\beta\cos\lambda\right)^2 - \sigma_P^2\right]$$
$$\times \cos\alpha\cos^2\beta\cos\lambda\, d\alpha\, d\beta\, d\lambda. \tag{10}$$