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Minimizing hydraulic resistance of a plant root by shape optimization

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ABSTRACT

Optimum plant root structure that minimizes a single root's hydraulic resistance to water-uptake is studied in this paper with the constraint of constant root volume. Hydraulic resistances under the slender body approximation and without such a limitation are considered. It is found that for large stele-to-cortex permeability ratio, there exists an optimum root length-to-base-radius ratio that minimizes the hydraulic resistance. A remarkable feature of the optimum root structure is that the optimum dimensionless stele conductivity depends only on a single geometrical parameter, the stele-to-root base-radius ratio. Once the stele-to-root base-radius ratio and the stele-to-cortex permeability ratio are given, the optimum root length-to-radius ratio can be found. While these findings remain to be verified by experiments for real plant roots, they offer theoretical guidance for the design of bio-inspired structures that minimize the hydraulic resistance for fluid production from porous media.

1. Introduction

Water-uptake from soil by a plant's root system has been studied extensively during the past seven decades due to its importance to plant biology and ecology [1–12]. At the root-system level, water-uptake by a plant is coupled to water transport in the soil by the Richard's equation [10]

$$\frac{\partial \theta}{\partial t} = \nabla \cdot [K \nabla p] - S, \tag{1}$$

where θ is the volumetric water content in the soil; $K = K(\theta)$ is the water conductivity in the soil which depends on the local water content; p is the water pressure in the soil pores which is also related to the local water content, $p = p(\theta)$; and S is a volumetric sink term representing the local root water-uptake. The sink term S, also known as the water extraction function [5], is the local water-uptake averaged over a large number of rootlets [10] and it characterizes the coupling between the plant's root water-uptake and the water content in the soil at a macroscopic level. At this large length scale, a single rootlet can be treated as infinitesimally small and the sink term S can be modeled as a point function given by

$$S = N(\mathbf{x}) \frac{p(\theta) - p_r(\boldsymbol{x})}{R}, \tag{2}$$

where $N(\mathbf{x})$ is the root density distribution (number of root per unit soil volume); R is the single root hydraulic resistance; $p(\theta) - p_r(\mathbf{x})$ is the difference between the water pressure in the soil and the water pressure in the root; and $\left(p(\theta) - p_r(\mathbf{x})\right)/R$ is the water-uptake by a single root. The root pressure $p_r(\mathbf{x})$ is controlled by the hydrodynamics in the root system. The hydraulic resistance of a single root R is a property of the root which depends on the root's internal structure; and it must be obtained from a mesoscopic scale analysis of a single root hydraulics.

A plant root has a composite structure with a core called stele and an annulus called cortex (Fig. 1). Water flow in the stele is through the longitudinal xylem tubes, which are responsible for siphoning water up to the tree leaves. Historically, a root has been modeled as an infinite long co-axial cylinder of the stele and the cortex when computing its hydraulic resistance [1–4,10,13]. Recently, Chen [14,15] has shown that the three-dimensional effect around the tip of a finite length root enhances the water-uptake rate and a root's hydraulic resistance is significantly lower than that predicted by the infinitely long cylinder model.

The objective of the present paper is to study the optimal structure of a single root that gives rise to a minimal hydraulic resistance to water-uptake under the constraint of prescribed root volume. For simplicity, the root is modeled as one-half of a prolate-spheroid with the stele as one-half of a confocal prolate-spheroid. The hydraulic resistance for such a model derived by Chen [15] shows that a root's hydraulic resistance decreases with the length of the root as well as the dimensionless conductivity of the stele. However, when the root volume is prescribed, increasing the root length leads to a decrease in the stele cross-sectional area, which

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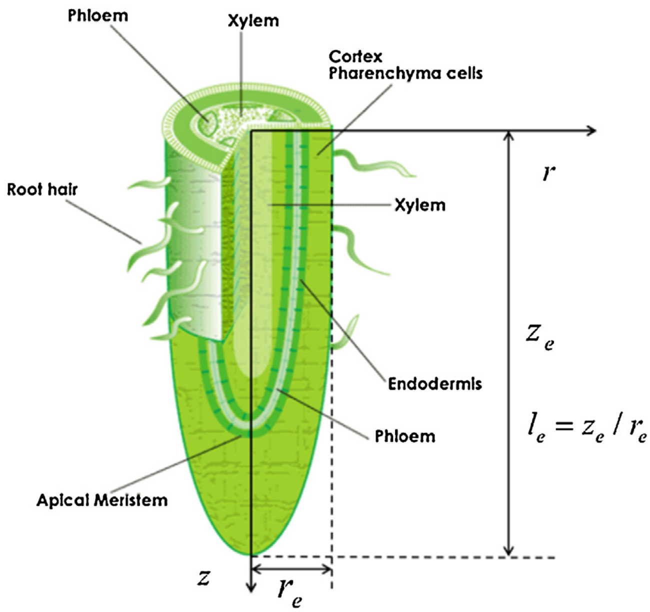

Fig. 1. A root modeled as a composite structure of two ellipsoids.

reduces the stele conductivity. Thus, there exists an optimal length-to-diameter ratio for the root that minimizes the root's hydraulic resistance. This optimal root structure is sought in the present study.

2. Hydraulic resistance of a single root in the limit of slender body approximation

The root surface is geometrically modeled as one half of a prolate-spheroid with the base of the root located at $z = 0$ (Fig. 1). Prolate spheroidal coordinates (ξ, η, φ) are used, and the Cartesian coordinates (x, y, z) are related to the prolate spheroidal coordinates by

$$x = L\sqrt{\xi^2 - 1}\sqrt{1 - \eta^2}\cos\varphi$$
$$y = L\sqrt{\xi^2 - 1}\sqrt{1 - \eta^2}\sin\varphi\,, \qquad (3)$$
$$z = L\xi\eta$$

where $1 \le \xi < \infty$, $0 \le \eta \le 1$, $0 \le \varphi \le 2\pi$; L is the focal distance. The root surface is described by $\xi = \xi_e$. The interface separating the stele from the cortex vessels is modeled similarly as one half of a prolate-spheroid confocal with the root surface, $\xi = \xi_0$ $(\xi_0 < \xi_e)$. $z_0 = L\xi_0$ is the length of the stele, and $z_e = L\xi_e$ is the length of the root. At the base of the root, $z = 0$, the radius of the stele is $r_0 = L\sqrt{\xi_0^2 - 1}$ and the radius of the root is $r_e = L\sqrt{\xi_e^2 - 1}$. The root's length-to-base-radius ratio is $l_e = z_e/r_e = \xi_e/\sqrt{{\xi_e}^2 - 1}$. The stele core and the cortex annulus occupy the regions $1 \le \xi \le \xi_0$ and $\xi_0 \le \xi \le \xi_e$, respectively. Gravity is negligible at single root scale.

Under the slender-body approximation [16], the hydraulic resistance of the root is given by [15],

$$R = \frac{\mu}{2\pi\kappa_C L} f(C_{sD}, \xi_0, \xi_e) \qquad (4)$$

where μ is the water viscosity; κ_C is the cortex permeability.C_{sD} is a dimensionless conductivity of the stele which measures the hydraulic conductivity of the stele relative to that of the cortex,

$$C_{sD} = \lambda(\xi_0^2 - 1) = \frac{\kappa_S r_0^2}{(\kappa_C L^2)}, \qquad (5)$$

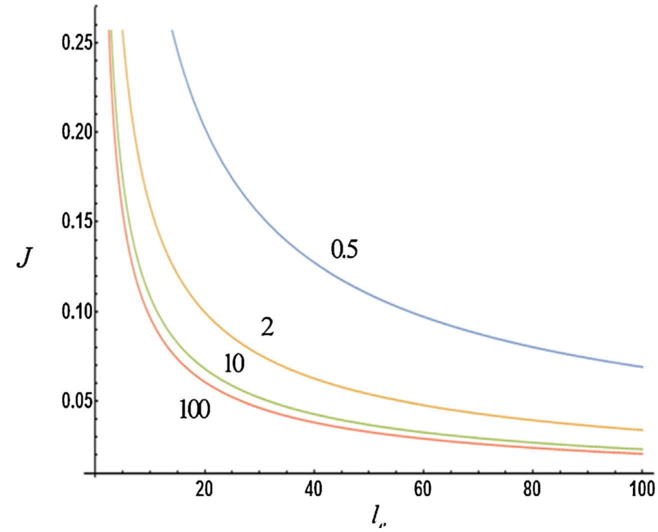

Fig. 2. $J(l_e, C_{sD}, \beta)$ vs l_e. C_{sD} value is shown in the figure. $\beta = 0.5$.

where κ_S is the permeability of the stele, and

$$\lambda = \frac{\kappa_S}{\kappa_C} \qquad (6)$$

is the ratio between the stele permeability and the cortex permeability.

3. Optimal root shape for minimum hydraulic resistance

The volume of the entire root is given by $V_e = 2\pi z_e {r_e}^2/3$. Thus,

$$L = \left(\frac{3V_e}{2\pi}\frac{1}{\xi_e({\xi_e}^2 - 1)}\right)^{1/3}. \qquad (7)$$

Therefore, the hydraulic resistance can be expressed as

$$R = \frac{\mu J(C_{sD}, \xi_0, \xi_e)}{2\pi\kappa_C\left(\frac{3V_e}{2\pi}\right)^{1/3}}. \qquad (8)$$

$J(C_{sD}, \xi_0, \xi_e)$ is listed in Appendix A. The ellipsoidal co-ordinate ξ_e that represents the root–soil surface can be expressed in terms of the length-to-base-radius ratio of the root,

$$\xi_e = \frac{1}{\sqrt{1 - 1/{l_e}^2}}. \qquad (9)$$

We also define a parameter β that represents the ratio between the base-radius of the stele and the base-radius of the root,

$$\beta = \frac{r_0}{r_e} = \frac{\sqrt{\xi_0^2 - 1}}{\sqrt{\xi_e^2 - 1}} < 1. \qquad (10)$$

In the following sections, we discuss the dependence of the hydraulic resistance on the shape of the root.

3.1. *Dependence of hydraulic resistance on l_e for constant conductivity C_{sD}*

When the volume V_e of the root is prescribed and the parameters C_{sD}, μ, κ_C and β are given, the hydraulic resistance depends only on the length-to-base-radius ratio l_e; and this dependence is described by the function $J(l_e, C_{sD}, \beta)$. For a fixed value of stele-radius to root-radius ratio β, a series of plots of J vs l_e for different values of the stele conductivity C_{sD} are shown in Fig. 2. These plots show that the hydraulic resistance decreases when the root length-to-base radius