Precision Engineering 40 (2015) 46–54

Contents lists available at ScienceDirect

Precision Engineering

journal homepage: www.elsevier.com/locate/precision

A parameter optimization framework for determining the pseudo-rigid-body model of cantilever-beams

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ARTICLE INFO

Article history:
Received 16 May 2014
Received in revised form 12 August 2014
Accepted 1 October 2014
Available online 17 October 2014

Keywords:
Compliant mechanisms
Pseudo-rigid-body model
Parameter optimization
Timoshenko beam theory

ABSTRACT

This paper focuses on developing a framework for determining the optimal pseudo-rigid-body (PRB) model of 2D cantilever beams. PRB models are commonly used in design and analysis of compliant mechanisms since they significantly reduce the number of degrees of freedom compared with the finite element approach. Although a number of PRB models are available in literature, there is not a unified method to determine the most suitable pseudo-rigid-body model for a specific application. In this work, we first study a modified Timoshenko beam equation which accommodates shear forces and axial deformation. The numerical solution to the Timoshenko beam equation provides a baseline for comparing various models. A novel concept of "PRB matrix" is proposed for representing topologies of all PRB models in a uniform way. The optimal set of kinematic parameters (characteristic lengths and spring constants) of PRB models are determined by minimizing the error of tip deflection and comparing with the solution of the Timoshenko beam equation. To validate this formulation, we compare the results with existing PRB models and obtained equivalent if not a more accurate set of PRB parameters. At last, a case study of a compliant slider mechanism is provided to demonstrate the accuracy of two PRB models in this particular application.

1. Introduction

Compliant mechanisms provide designers the option of replacing rigid-body linkages with flexible members. This lends intrinsic spring-like characteristics, which can be very useful for control, especially in robotics applications. Such robots provide great precision and control, and using varying stiffness and damping, the designers can achieve stability [1]. Compliant mechanisms are also widely used in micro-electro-mechanical-systems (MEMS) and other micro-scale applications [2]. However, design of compliant mechanisms can be a difficult process due to the nonlinearities arising out of their deflections. This places a great emphasis on the need to improve the methods of analysis. With a better understanding of behavior of compliant elements and enhancing the tools for analysis, there will be a significant advancement in the use of these mechanisms.

Compliant mechanisms have been studied in great detail over the past couple of decades. Over the course of this time, different methods have been suggested for the analysis of compliant mechanisms. One approach has been to use topological methods, which use continuum synthesis [3–5]. Another method is the beam constraint model proposed by Awtar et al. [6]. Although it provides a closed form solution and takes into account the effects of load stiffening and other nonlinearities, it assumes small deflections. A third approach, which will be discussed in this paper, is to use kinematic models for analysis and design. This approach replaces flexible elements with rigid-body replacements, which simplifies the equations. Such a kinematic model is called a pseudo-rigid-body (PRB) model [7,8]. Since rigid-body kinematics are well studied, the PRB approach is usually more intuitive for the purpose of analysis and design. However, care needs be taken to ensure that the model is accurate in replicating the behavior of the flexible members.

Most of the early work on PRB models used only one independent revolute joint. Even the ones with more than one joint were symmetrical [9]. Dado [10] developed a variable parametric model. However, these models are load dependent, which makes them inconvenient for design and synthesis in which the load to the beam tip is unknown. Su [11] suggested a load independent 3R model in which the parameter values do not change with the loading conditions, which was found to be highly accurate for beam like structures. Chen et al. [12] used a particle swarm optimizer to suggest better values for the parameters. But this model may be too complex for simple elements, leading to increased computational cost. Recently, Yu et al. [13] proposed a 2R model that reduces the

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http://dx.doi.org/10.1016/j.precisioneng.2014.10.002
0141-6359/

$[\Omega]$	General PRB matrix
$[J]$	Jacobian matrix
α_x	Dimensionless force in the x-direction
α_y	Dimensionless force in the y-direction
β	Dimensionless moment in the z-direction
Δ	Deflection of extension spring
γ	Initial characteristic length of the segment
κ	Shape factor of cross section for shear
ν	Poisson's ratio
ϕ	Tip deflection angle
θ	Deformation of torsion spring
$\vec{\tau}$	Internal forces and moments in the PRB
$\vec{F}$	Load on the beam tip
A	Area of cross section
a	x coordinate of beam tip
b	y coordinate of beam tip
E	Elastic modulus of the material
G	Shear modulus
I	Second moment of area
K_θ	Stiffness of torsion spring
k_θ	Dimensionless parameter for stiffness of torsion spring
K_{ex}	Stiffness of extension spring
k_{ex}	Dimensionless parameter for stiffness of extension spring
L	Length of the beam
N	Number of segments in PRB model
t_0	Thickness of the beam
w_0	Width of the beam

complexity of the 3R PRB model while maintaining a similar level of accuracy. Another method for using PRB models with revolute joints was suggested by Pei et al. [14]. Some models also use prismatic joints with extension springs, such as the one presented by Saxena and Kramer [15] or Vogtmann et al[16].

As mentioned in the previous paragraph, one of the major drawbacks of most of these models is the dependence of the parameters on the loading direction. Although this is useful for static analysis, where the loads and boundary conditions are known, it makes these models difficult to use for dynamic simulations or mechanism synthesis. Load independent models are more useful for these cases. Since compliant mechanisms usually have a limited range of motion, the models only have to be accurate within the expected range of loading or deformation. It may also be important to have model parameters based on the type of deformation expected.

The required accuracy and kinematics of the PRB model also depend on the application. It may be useful to develop a model on a case to case basis after studying expected deformation, required accuracy and other application requirements. With this in mind, we can make a strong argument for a general PRB model that can adapted for each case. The model can be modified to be simple or complex based on the demands of the application. Since the design process generally involves multiple evaluations, a simple model with minimum parameters would be suitable for this. For analysis, the user may be better served having a complex model that captures all the characteristic behavior of the compliant element.

The rest of the paper will follow the development of a general PRB model, which can be adapted to different scenarios. Many possible permutations of this model will be looked at, and the accuracy and complexity of each of them tabulated. This methodology will also be used to prove the validity of existing PRB models and suggest minor modifications that lead to better results. The concept of a PRB matrix will be introduced, along with a discussion on its

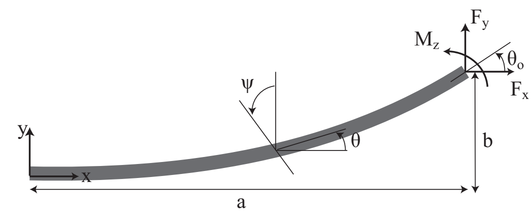

Fig. 1. Cantilever beam with end loads.

interpretation. We will also look at a direct approach to finding the values of the parameters based on an optimization algorithm. Finally, a simple procedure will be suggested that leads the user to the most effective PRB model for each situation based on available data. For validation, a case study involving a simple compliant slider mechanism will be presented. But first, a numerical approach to solving a beam equation for highly elastic materials based on the Timoshenko beam thoery will be discussed.

2. Timoshenko beam model

While working with soft polymers with rubber-like properties, it was seen that axial deformation contributes significantly to the loading behavior of the part. Many soft joints have been fabricated using these materials with the Shape Deposition Manufacturing (SDM) [17] approach. In order to improve analysis of these joints, it was necessary to take into consideration the elongation and Poisson's effect. The parameters of the PRB model are determined based on the expected behavior of the compliant elements. There are many ways to ascertain the deformation of the compliant members, such as experiments or FEA simulations. Another approach is to model them as beams and use beam theory to determine the deflection and orientation, for which a suitable beam model is to be selected. This section describes a modified Timoshenko beam model we used for the calculations.

Timoshenko [18] developed a theory for the analysis of short beams which takes into account shear deformation and rotational inertia effects, which makes it a comprehensive approach for small applications. Generally, the model is derived in terms of the horizontal variable x, which means that it is valid only for small deformations. However, for rubber like materials which are used extensively in compliant mechanisms, large deformation analysis is necessary. To deal with this, the independent variable x can be changed to s, which is the length along the curve of the beam. The model also takes into account the Poisson effect, and includes the change in cross section that occurs due to the same. See Fig. 1.

Consider a cantilever beam with of length L and a rectangular cross section of width w_0 and thickness t_0. When it is subject to a general F_x, F_y and M_z at its free end, it is deformed to a shape with $x(s)$, $y(s)$, $\theta(s)$ defining the coordinates and the slope angle of the beam at any point $0 \le s \le L$ on the beam.

The axial force, shear force and bending moment of the beam are given by $P(s)$, $V(s)$ and $M(s)$, written as

$$P(s) = F_y \sin\theta(s) + F_x \cos\theta(s) \tag{1}$$

$$V(s) = -F_y \cos\theta(s) + F_x \sin\theta(s) \tag{2}$$

$$M(s) = M_z + F_y(a - x(s)) - F_x(b - y(s)) \tag{3}$$

The axial elongation of the beam is represented using the following two equations.

$$\frac{dx}{ds} = \left(1 + \frac{P(s)}{EA}\right)\cos\theta(s) \tag{4}$$

$$\frac{dy}{ds} = \left(1 + \frac{P(s)}{EA}\right)\sin\theta(s) \tag{5}$$