Mechanics Research Communications 73 (2016) 140–144

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

The coefficient of normal restitution for the Hertzian contact of two rough spheres colliding in a viscous fluid

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ARTICLE INFO

Article history:
Received 23 October 2015
Received in revised form 17 February 2016
Accepted 28 February 2016
Available online 8 March 2016

Keywords:
Particle collisions
Coefficient of restitution
Hertzian contact
Lubrication force

ABSTRACT

We use previous theoretical results for the added mass, history and lubrication forces between two spheres colliding in a fluid with viscosity ν to investigate the effect of viscous dissipation on the coefficient of restitution during contact. We assume that the mechanical interaction is governed by Hertzian mechanical contact of small duration τ and that the minimum approach distance between particles is approximately equal to the height σ of surface micro-asperities. A non-dimensionalization of the equation of motion indicates that the contact dynamics is governed by two parameters – the ratio ϵ of the surface roughness σ and the sphere radius a, and a contact Stokes number defined as $St_c = \sigma^2/\nu\tau$. An asymptotic solution of the equation of motion in the limit of small ϵ/St_c is used to obtain an explicit expression for the coefficient of restitution during contact and the latter is compared with estimates based on numerical solutions of the non-linear equation of motion.

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1. Introduction

The coefficient of restitution e is a key parameter describing the dynamics of particle collisions and the dissipation of kinetic energy in granular and particle-laden flow. While the coefficient of restitution for collisions in viscous fluids has been the subject of many investigations, most of the attention has been directed to the hydrodynamic damping during the approach of the particles and the question of viscous dissipation during mechanical contact has received little attention. Experimental investigations [1–4] have found that lubrication forces generated by the drainage of the viscous fluid film in the gap of approaching spheres results in a significant damping of the sphere momentum (and a reduction of e) for collisions with low impact Stokes number $St = (m_* W_\infty)/(6\pi\rho\nu a_*^2)$, where $a_* = a_1 a_2/(a_1 + a_2)$ and $m_* = m_1 m_2/(m_1 + m_2)$ are the sphere reduced radius and mass, W_∞ is the "impact" velocity (at separations where lubrication effects are negligible), and ν and ρ are the fluid viscosity and density. The laboratory experiments of sphere-wall collisions in a viscous fluid [1–3] have focused on estimating an effective coefficient of restitution e_{wet} which includes the strong hydrodynamic interaction of the particles prior to making contact. The laboratory experiments have established that e_{wet} decreases at low impact velocities and is a well-defined function of the collision Stokes number St, such that e_{wet} becomes zero below a critical $St \approx 10$. Our focus here is on the contact dynamics during viscous collisions of small particles in industrial and marine sediment transport flows where the impact Stokes numbers are typically less than 100 and where the coefficient of restitution is reduced significantly below $e = 1$.

Existing estimates of e_{wet} based on integrating Newton's second law for the particle motion (e.g Barnocky and Davis [1]) typically neglect the hydrodynamic forces during mechanical contact, and assume that all the viscous dissipation occurs before and after the particles make contact. For mechanical contact characterized with a coefficient of restitution e_{dry}, a simple prediction

$$e_{wet} = e_{dry} - (1 + e_{dry})St^{-1} \log\left(\frac{\sigma_0}{\sigma}\right) \tag{1}$$

was obtained in Joseph et al. [3] by integrating Newton's equation for the motion of a sphere subject to lubrication force from some small initial separation σ_0 to a minimum separation distance corresponding to the size σ of surface microasperities. It was found that the model prediction is consistent with the observed dependence of e_{wet} on W_∞. However, we note that the comparisons of Eq. (1) with measurements (e.g. Joseph et al. [3], Yang and Hunt [4]) utilize $e_{dry} < 1$ which implies that additional dissipation occurs during contact. Marshall [5] extended the e_{wet} model of Barnocky and Davis by including viscous damping during contact based on an approximate model for the viscous corner flow associated with the lateral expansion of the contact region. The purpose of the present

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http://dx.doi.org/10.1016/j.mechrescom.2016.02.017
0093-6413/Published by Elsevier Ltd.

paper is to investigate the effect of hydrodynamic dissipation during contact based on an asymptotic solution for the viscous gap flow [6] and to estimate the corresponding reduction of the coefficient of restitution during Hertzian contact, hereinafter denoted by e_{con}, below the theoretical value $e_{con} = 1$. The goal of this investigation is to determine the dependence of the coefficient of restitution on the particle roughness, the impact velocity, and the material properties of the solids and the interstitial fluid.

2. Coefficient of restitution in dry contacts

The modeling of collisions begins with understanding the purely mechanical interaction during dry contacts where hydrodynamic forces are negligible. Extensive laboratory studies have demonstrated (see Goldsmith [7]) that the Hertzian elastic theory of contact is adequate for describing the dynamics of the mechanical contact during moderate collisions of hard spheres in air where hydrodynamic dissipation is negligible. It is well-known that the Hertzian force between two spherical particles is proportional to the 3/2-power of the overlap distance Δ at the point of contact [7]

$$F_H = -k_H \Delta^{3/2}, \tag{2a}$$

where the Hertzian stiffness parameter k_H is related to the Young's moduli E_1 and E_2, and the Poisson ratios λ_1 and λ_2 of the two solids as follows

$$k_H = \frac{2}{3} E_* a_*^{1/2}; \tag{2b}$$

$$E_* = \frac{2}{\frac{1-\lambda_1^2}{E_1} + \frac{1-\lambda_2^2}{E_2}}; \tag{2c}$$

For given impact velocity $\overline{W}$, the maximum Hertzian deformation of the particle boundaries is

$$\delta_H = \overline{W} \tau_* \tag{3}$$

where τ_* is the Hertzian time scale

$$\tau_* = \left[\frac{25 m_*^2}{16 \overline{W} k_H^2} \right]^{1/5}. \tag{4}$$

When the impact velocity is greater that a critical value (Johnson [8])

$$v_{yield}^2 \approx \frac{106 a_*^3 Y^5}{(m_* E^4)} \tag{5}$$

related to the yield strength Y of the material, most of the energy dissipation during dry collisions results from plastic deformation; for most materials, this critical velocity is on the order of a few cm/s. When the impact velocity is less than the critical value v_{yield}, only viscoelasticity of the solid is thought to dissipate energy in dry collisions. The general form of the total mechanical force in such viscoelastic collisions is obtained by adding a dissipative force proportional to the strain rate $\dot{\Delta}$ (Kuwabara and Kono [9], Schafer et al. [10], Brilliantov et al. [11], Falcon et al. [12]) to the Hertzian force (2a),

$$F = -k_H \Delta^{3/2} - \mu \dot{\Delta} |\Delta|^{\gamma}, \tag{6}$$

where μ is a parameter related to the internal frictional dissipation in the solid.

The dissipation in such viscoelastic collisions can be analyzed by non-dimensionalizing the equation of motion for the particle pair

$$m_* \frac{dW}{dt} = -k_H \Delta^{3/2} - \mu \dot{\Delta} |\Delta|^{\gamma}, \tag{7}$$

using the impact velocity $\overline{W}$ and the Hertzian time scale to define the non dimensional velocity $w = W/\overline{W}$, non-dimensional time $\tau = t/\tau_*$ and non-dimensional displacement $\delta = \Delta/(\tau_* \overline{W})$. It is then found that the resulting non-dimensional equation

$$\frac{dw}{d\tau} = -\frac{5}{4}\delta^{3/2} - \frac{\mu \tau_* \left(\overline{W}\tau_*\right)^{\gamma}}{m_*} \dot{\delta} |\delta|^{\gamma}, \tag{8}$$

depends on a single dimensionless parameter $\mu \tau_* m_*^{-1} (\overline{W} \tau_*)^{\gamma}$ that represents the ratio of the restoring and dissipative mechanical forces. Multiplying by $w = \dot{\delta}$ and integrating the result during the contact period in the limit of weak dissipation $\mu \to 0$, it is possible to obtain an analytical expression for the coefficient of restitution (see Kuwabara and Kono [9], Falcon et al. [12], also Ray et al. [13])

$$e_{dry} = 1 - \frac{4}{5} \mathcal{B}\left(\frac{3}{2}, \frac{2(\gamma+1)}{5}\right) \frac{\mu \tau_*}{m_*} \left(\tau_* \overline{W}\right)^{\gamma}, \tag{9}$$

where $\mathcal{B}$ is the Beta function. The case $\gamma = 1/2$ corresponds to a viscoelastic solid and predicts a dry coefficient of restitution that decreases weakly with the impact velocity proportional to $-\overline{W}^{1/5}$. Verification of the prediction (9) for $\gamma = 1/2$ has been difficult due to lack of measurement data for μ which is related to the solid viscosity coefficients.

For rough surfaces, the range of plastic dissipation extends to much lower velocity as the particle radius in the yield condition (5) is replaced by the effective asperity curvature radius. The Hertzian force model (2a) is valid for rough surfaces only at high loads. At low loads, the presence of surface roughness tends to increase the effective contact area (relative to Hertzian contact of smooth surfaces) and leads to an approximately linear elastic force [14]. The detailed investigation of such complex multi-asperity contact models (see [15] for a recent review) is beyond the scope of the present paper where the focus is on the viscous dissipation due to the interstitial fluid. We will illustrate the effect of viscous dissipation using the Hertzian contact model which is commonly assumed to be valid over the entire range of loads in numerical models of particle-laden flows. The investigation of slow viscous collisions in the following section will show that hydrodynamic effects can give rise to a dissipative term similar to the second term in Eq. (6).

3. Coefficient of restitution in viscous contacts

The main obstacle to evaluating the effect of hydrodynamic forces during contact has been the lack of theoretical model or measurements of these forces. For that reason, numerical modelers (e.g. Derksen and Sundaresan [16], Simeonov and Calantoni [17]) have used various ad hoc assumptions to limit the magnitude of the lubrication force during contact. Here, we should point that classic theories for the lubrication force (e.g. Cooley and O'Neill [18]) are based on the steady Stokes equation and, as such, are not appropriate to describe the hydrodynamics during mechanical contact when the particle velocity changes very rapidly and imparts large accelerations on the surrounding fluid.

The unsteady hydrodynamics of colliding particles was recently investigated in [6], where following Barnocky and Davis [1], the particles were assumed to make contact through microasperities of height σ so that the gap between the particles remains finite during contact. This asperity contact model should be contrasted with the elastohydrodynamic theory of Davis et al. [19] for the viscous collision of smooth particles where the minimum separation between the particles δ_{EHL} becomes so small that the fluid pressure in the gap generated by the squeezed flow becomes sufficiently large to transmit the mechanical stresses without actual contact. For low impact collisions, the minimum approach distance δ_{EHL} is typically much smaller than real particle roughness which is in the