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The initial value problem for the mathematical model of a multiple-input multiple-output flexible parallel mechanism

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ABSTRACT

The cable-net support structure of the Five-hundred-meter Aperture Spherical Telescope (FAST) is a multiple-input multiple-output (MIMO) flexible parallel mechanism. The mathematical model of the cable-net structure is set up with the catenary equation, which produces a large number of non-linear equations. Finding the initial value plays an important role in the working state of the FAST. In this paper, a novel method is proposed to solve the equations of the model. This method uses the solution of the $(n-1)$-th layer cable-net as the initial value of the n-th layer cable-net. The n-layer cable-net equations are then solved simultaneously and the tension of each cable and the coordinate of each node in the cable-net can be obtained. With the introduction of the symmetry of cable-net, the number of unknown parameters in the equations is reduced to about one-tenth of the whole cable-net. The initial value of the cable-net support structure of the FAST is provided to show the validity of the method. The results of this study are of practical value for the FAST's new type of flexible cable driving mechanism design and control method.

1. Introduction

Based on approval from the National Development and Reform Commission, Chinese scientists have put forward an innovation design for the largest single dish radio telescope [1,2], the Five-hundred-meter Aperture Spherical Telescope (FAST) in a Karst depression in GuiZhou province [3]. A 3-D simulated image of the FAST is shown in Fig. 1. With the preliminary research completed in the past few years, the FAST project is officially one of China's major scientific projects for the national "11th Five-Year Plan". The main ring beam was built on the last day of 2013 as shown in Fig. 2.

The radius of the dish is 250 m and the main reflector, a spherical dish, has a radius of 300 m when it is in its tension state. A paraboloid of revolution shaped dish is formed by adjusting the length of the driving cable with the diameter of opening of 300 m, as shown in Fig. 3. The direction of the paraboloid of revolution is facing the tracking target when the observation starts. The feed cabin can detect the signals when it moves on the focal surface under the control of the supporting cables. According to the working principle of the FAST, the cable-net support structure of the FAST is a multiple-input multiple-output (MIMO) flexible parallel mechanism. The input is the length of the driving cable and the output is the paraboloid of revolution on the spherical dish when the FAST is in its working state.

The supporting structure of the main reflector surface is made of cables, as shown in Fig. 4. The lightweight cable-net structure reflector for the FAST is a new driving mechanism. Because of its efficiency and esthetics, the cable-net structure has been widely used in large-span roofs, bridges, and large-diameter antennae etc. [4,5]. Presently, there are some methods for solving the cable-net structure problem. In some of the papers [6,7], the force density method was introduced and the main feature of this

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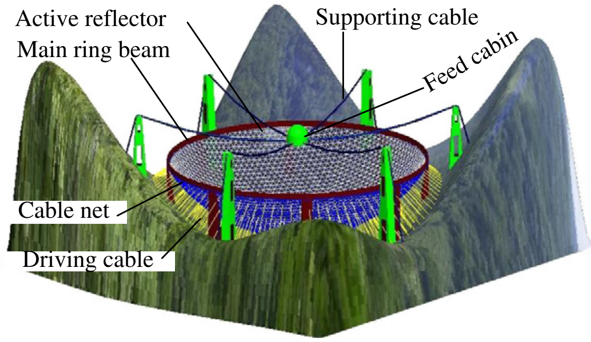

Fig. 1. 3-D simulated model of the FAST.

method lies in prescribing a force density coefficient and a force-to-length ratio for each cable element. The dynamic relaxation method is applied to the elastic-plastic bending of circular plates in the large deflection [8–10] and nonlinear finite element methods [11–14]. All of these methods only provide the approximate solution. Furthermore, the nonlinear finite element method has been adopted to research the cable-net supporting structure of the FAST when it is under the tension state and working state [15–17]. By virtue of the method, the working state of the FAST is achieved by repeating the inverse iteration method to solve for the length of the drag cables. In this paper, the cable-net supporting structure is treated as a MIMO flexible parallel mechanism. The mathematical model of the cable-net supporting structure is built with the catenary equation and the working state of the FAST can be obtained by solving the nonlinear equations in real time and therefore the length of the drag cables can be solved through a single step calculation. In previous work [18], the catenary equation is used to solve a simple cable-net structure and because the number of the catenary is smaller, it does not require the initial value of the catenary equations. The supporting cable-net of the FAST, however, is a very a large structure, and the mathematical model has a large number of non-linear equations. Therefore, the initial value is very important for solving the non-linear equations. A new layer-wise substitution method is different from other known methods. The method proposed is based on the exact solution of the catenary equations. The use of the analytic expression of the catenary to solve or formulate other complex problems has been used previously by some scholars [19–24]. The differences between the aforementioned models and herein proposed one are as follows: there are many more parameters of the herein model than the aforementioned models, and furthermore, the whole cable-net is controlled by the driving cables of each node.

The cable-net is a spherical dish when it is in the tension state. The type of mesh is a geodesic mesh and the distance between two adjacent nodes is about 11 m. The position of each node can be solved in the global frame. According to the characteristics of the cable-net structure, the cable-net is divided into 31 layers from the center point to its boundary. The division method of the cable-net will be described in detail in the following section. Since the practical length of the main cable is unknown (the main cable refers to the cable between two adjacent nodes), the chord length between two adjacent nodes on the spherical dish can be seen as the main cable length. Fig. 4 illustrates the cable-net support structure of the FAST.

Fig. 2. Main ring beam of the FAST.